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AMALY PROGRAMMALAR PAKETI

Ýokary okuw mekdepleriniň talyplary üçin okuw gollanmasy

Türkmenistanyň Bilim ministrligi tarapyndan hödürlenildi

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Bu okuw gollanmasynda häzirki zaman kompýuterlerinde giňden ulanylýan amaly programmalar paketleri we olary döretmekligiň hem-de ulanmaklygyň aýratynlyklary barada maglumatlar getirilýär. Gollanma ýokary okuw mekdepleriniň talyplaryna, informatika mugallymlaryna we giň okyjylar köpçüligine niýetlenen.

Giriş.

Hormatly Prezidentimiz Gurbanguly Berdimuhamedowyň atalyk aladalary netijesinde bedew badynda durmuşa geçirilýän täze özgertmeler we beýik galkynyşlar zamanasynda Türkmenistanyň ylym-bilm ulgamy düýpli ösüşi başdan geçirýär. Gurulýan täze bilim ymaratlary iň kämil multimedia we interaktiw tehnologiýalar bilen enjamlaşdyrylýar. Bular bolsa, esasan ösen kompýuter tehnologiýalary bilen baglanşyklydyr. Şonuň üçin kompýuterleriň enjam we programma düzüm böleklerini ulanmaklygy başarmak, kompýuterdäki programmirleme usullaryny öwrenmek häzirki döwruň hünärmenlerine, şol sanda talyp-ýaşlara bildirilýän esasy talaplaryň biridir.

Köp amaly meseleler kompŷuterlerde çözülende şu yzygiderli işleri berjaý etmeli bolýar: 1) Meseläniň çözülişiniň algoritmini düzmek; 2) Algoritme esaslanyp, käbir algoritmik dilde programma düzmek; 3)Programmany kompýutere girizmek, ony testirlemek we netije almak.

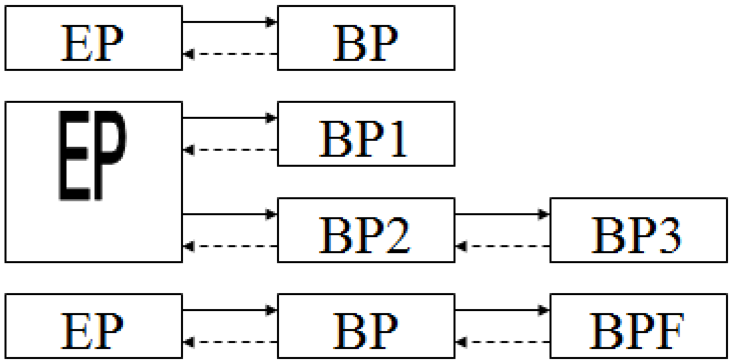
Kompýuterde programma düzlende köplenç programmalaryň dürli ýerlerinde şol bir operatorlar yzygiderliklerini ýerine ýetirmek gerek bolup durýar. Köplenç halatlarda üýtgeýän ululyklaryň dürli-dürli bahalarynda operatorlar yzygiderlikleri ýerine ýetirilýär bu ýagdaŷlarda operatorlar yzygiderligi bölek programma görnüşinde ýazylýar Beýsik dilinde bölek programma ýüzlenmeklik esasy programmada

GOSUB N

operatory ýazmaklyk bilen amala aşyrylýar. Bu ýerde N-bölek programmalaryň birinji operatorynyň nömeri bolup durýar. Bölek programma N-nji setir bilen başlanýan operatorlar yzygiderligi bolup iň soňky operatory

RETURN

bolmalydyr. Return operatory esasy programmanyň gosub N operatoryndan soňky operatora geçmek bilen esasy programma işini dowam etdirýär. Esasy programma (EP) bilen bölek programmanyň (BP) we bölek programma funksiýanyň (BPF) arasyndaky hereketleri aşakdaky görnüşde görkezmek bolar:



Şeýlelik bilen EP bilen BP-da ulanylýan üýtgeän ululyklary 4-sany topara bölmek bolar.

1) girizillýän
2) çykarylýan
3) içki
4) başgalar

Girizilýan ululyklar EP özüniň hakyky bahasyny alyp BP ol bahalar ulanylýar.
Çykarylýan ululyklar BP netijesi bolup durýar.Netijede girizilýän we çykarylýan ululyklar EP, BP bilen informasiýa çalşygyny amala aşyrýar. Içki üýtgeýän ululyklary BP girizilýän we çykarylaýn ululyklardan başgalary bolup hysmat edŷär. Başgalar-EP girizlýän we çykarylýan ululyklardan başgalary. Bu ululyklaryň arsynda baglanyşyklaryny käbirini belläp geçeliň:

1) BP ýüzlenilmänkä girizilýän ululyklaryň hakyky bahasyny bermeli.
2) Içki we başgalar üýtgeýän ululyklaryň gabat getirmezliklige çalyşmaly.
3) Bölek programma diňe ýüzlenilen halatynda işlemegini gazanmaly.

1. Amaly programmalar paketiniň umumy gurluşy.

1) Goý *5!, 6!, 7!, 8!* Bahalary tapmaly bolsun

```
Cls
Rem 5!, 6!, 7!, 8! {Hasaplamak}
For j=5 to 8
N=j
Gosub 1000
Print K
Next j
Stop
1000 Rem n! Has BP
Rem giriz ulylyk:N
Rem cykar ululyk:K
Rem icki ululyk:I
K=1
For I=1 to n
K=K*I
Next I
Return
```

2) Utgaşdyrma sanynyň bahasyny tapmaklygyň programmasyny düzmeli

$$C_n^m = \frac{n!}{m!(n-m)!}$$

```
Cls
Print "N,M-giriz"
Input N,M
L=N: gosub 1000:c1=P
L=N: gosub 1000:c2=P
L=N-P: gosub 1000
```

```
C=C1/(C2*p)
Print C
Stop
1000 Rem BP
P=1
For i=1 to l
P=P*i
Next i
Return
```

3) Kesimi deň ýarpa bölmek usulyny ulanyp

$$\cos\frac{2}{x}-2\sin\frac{1}{x}+\frac{1}{x}=0$$

Deňlemäniň [1,2] kesimde $\varepsilon=10^{-4}$ takylykda çözmeli

```
Cls
Def FNZ(x)=cos(2/x)-2*sin(1/x)+1/x
Print  "A,B,E-giriz"
Input A,B,E
Gosub 1000
Print x1
Stop
1000 A1=A:B1=B:F1=FNZ(A1)
1009 X1=(A1+B1)/2:F2=FNZ(x1)
If F2=0 Then goto 1014
If F1*F2<0 Then 1013
A1=X1:F1=F2: goto 1015
1013 B1=X1: goto 1015
1014 A=X1:B1=X1
1015 IF B1-A1>EPS Then goto 1009
Return
```

Matrisalaryň üstünde geçirilýän amallar üçin programmalaryň düzüliş usullary

Matrisa bu ikiölçegli massiw

$$A=\begin{pmatrix} a_{11} & a_{12} & .... & a_{1n} \\ a_{21} & a_{22} & .... & a_{2n} \\ ..... & ..... & .... & ..... \\ a_{m1} & a_{m2} & .... & a_{mn} \end{pmatrix}$$

bu ýerde matrisanyň ölçegi mxn, ýagny setir we sütün sanlarynyň köpeldilmagine aýdylýar. Kwadrat matrisa m=n bolanda bolýar, ölçegi $n^2=n*n$

1) Matrisany girizmek

Matrisany girizmekden öňürti onuň ölçegini girizmeli. Ol ***DIM A(M,N)*** görnüşde berilýär. Matrisany girizmeklik köplenç halatlarda setirler boýunça amala aşyrylýar

```
Cls
dim A(M,N)
Input ''M,N-giriz''M,N
For i=1 to M
For j=1 to N
Print "a("i ","j ")="
Input a(i,j)
next j
Next i
End
```

2) Diogonal matrisa

$$A=\begin{pmatrix} a_{11} & 0 & .... & 0 \\ 0 & a_{22} & .... & 0 \\ ..... & ..... & .... & ..... \\ 0 & 0 & .... & a_{mn} \end{pmatrix}$$

```
Cls
dim A(M,N)
Input ‘’M,N-giriz’’M,N
For i=1 to M
For j=1 to N
IF i<>j Then 60
Input a(i,j):goto 70
60 a(i,j)=0
70 next j: next i
End
```

3) Birlik matrisa- diogonal matrisanyň özgerdilen görnüşi, ýagny A(i,j)=1
haçanda i=j bolanda

```
Cls
dim A(M,N)
Input ‘’M,N-giriz’’M,N
For i=1 to M
For j=1 to N
IF i<>j Then 60
a(i,j)=1:goto 70
60 a(i,j)=0
70 next j: next i
End
```

4) Nul matrisa-başgaça arassalanan massiw ýagny a(i,j)=0

```
Cls
dim A(M,N)
Input ‘’M,N-giriz’’M,N
For i=1 to M
For j=1 to N
A(i,j)=0
next j:Next i
End
```

5) Konstantalar bilen amallar

```
Cls
Input n
dim A(N,N)
For i=1 to n
For j=1 to n
Input A(i,j)
A(i,j)= A(i,j)*X
next j:Next i
End
```

6) Transponirlenen matrisa A^T diýip sütüni setir bolup hyzmat edäýn kwadrat matrisa aýdylýar.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \qquad A^T = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

Bu ýerden görnüşi ýaly A,A^T matrisalaryň diogonal elementleri birmeňzeşdirler, galan elementlerini $a_{ij}^T = a_{ij}$, $j \neq i$ görnüşde hasaplanýar.

```
Cls
Input n
dim A(N,N)
For i=1 to n
For j=1 to n
Input A(i,j): next j:Next i
For i=1 to n:For j:=i+1 to n
c=a(i,j):a(i,j)=a(j,i):a(j,i)=c
next j:Next i
End
```

7) Goşmak we aýyrmak

Iki sany A we B ölçegleri MxN bolan matrisalary goşmak we aýyrmak $C_{ij} = a_{ij} + b_{ij}$ *i=1,2,...,m; j=1,2,...,n;* formula arkaly amala aşyrylýar

8) Matrisalary köpeltmek

m*n ölçegli A matrisany N*L ölçegli B matrisa köpeltmek

$$C_{kj} = \sum_{i=1}^{n} a_{kj} b_{jk} \quad j=1,2,...,l;\ k=1,2,...,m$$

formula arkaly amala aşyrylýar. Netije C m*l matrisa bolar.

```
Cls
Input m,n,l
dim A(M,N), B(N,L),C(N,L)
For i=1 to M:For j=1 to N
Input a(i,j)
next j: next i
For i=1 to N:For j=1 to L
Input b(i,j)
next j: next i
For k=1 to M:For j=1 to L:S=0
For i=1 to N S=S+A(k,i)*B(i,j)
 next i
C(k,j)=S:Next j:Next k
End
```

9) Kwadrat A matrisanyň kesgitleýjisi

2*2 matrisa üçin

$$D = a_{11}a_{22} - a_{21}a_{12}$$

3*3 matrisa üçin

$$D = a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{12}a_{23}a_{31} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{21}a_{12}a_{33}$$

n*n ölçegli matrisanyň kesgitleýjisini hasaplamak üçin Gaus usulyny ulanmak bolar. Onda matrisany özgertmek arkaly üçburçlyk görnüşde aşakdaky formula arkaly getirilýär:

$$a_{ij}^{(k)} = a_{ij}^{(k-1)} - a_{ik}^{(k-1)} \cdot \frac{a_{kj}^{(k-1)}}{a_{kk}^{(k-1)}} \quad \text{k=1,2,..,(n-1),}$$

$$a_{kk}^{(k-1)} \neq 0$$

Matrisanyň kesgitleýjisini özgerdilen matrisanyň hemme diogonal elementleriniň köpeltmek hasylyna aýdylýar.

```
Input n :Dim a(n,n)
For i=10 to n-1 : For j=0 to n-1
Input a(i,j):Next j:Next i
p=0:z=1, d=1:For k=0 to n-2
E=0 For i=k to n-1
For j=k to n-1:IF abs(E)>=ABS(A(i,j)) 90
E=A(i,j):B=i:c=j: Next j:Next i
90 If k=b Then 120
For j=k to n-1:S=A(k,j)
A(k,j)=A(b,j):A(b,j)=S:Next j
z=-z:If k=c Then 150
For i=k to n-1
S=A(i,k):a(i,k)=a(i,s)
A(i,s)=s:Next i:z=-z
150 for i=k+1 to n-1: G=a(i,k)/a(k,k)
For j=k to n-1:a(i,j)=a(i,j)-G*A(k,j)
Next j:Next i:Next k
For i=0 to n-1 :d=d*a(i,i)
next i:d=d*z
Print d
End
```

10) Ters matrisa-A^{-1} diýip berlen A matrisa bilen köpeltmek hasyly birlik matrisa berýän matrisa aýdylýar.

A matrisa ýüzlenmek diýip onuň ters matrisasyny A^{-1} tapmak diýmekdir

$$A \cdot A^{-1} = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots\dots & \dots\dots & \dots & \dots\dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix} \cdot \begin{vmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots\dots & \dots\dots & \dots & \dots\dots \\ x_{n1} & x_{n2} & \dots & x_{nn} \end{vmatrix} = \begin{vmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots\dots & \dots\dots & \dots & \dots\dots \\ 0 & 0 & \dots & 1 \end{vmatrix}$$

A we A^{-1} köpeldip n sany n^2 sany näbellilerden durŷan ulgamy alarys.

$$\begin{cases} a_{11}x_{11} + a_{12}x_{21} + \dots + a_{1n}x_{n1} = 1 \\ a_{21}x_{11} + a_{22}x_{21} + \dots + a_{2n}x_{n1} = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}x_{11} + a_{n2}x_{21} + \dots + a_{nn}x_{n1} = 0 \end{cases} \qquad (1)$$

$$\begin{cases} a_{11}x_{12} + a_{12}x_{22} + \dots + a_{1n}x_{n2} = 1 \\ a_{21}x_{12} + a_{22}x_{22} + \dots + a_{2n}x_{n2} = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}x_{12} + a_{n2}x_{22} + \dots + a_{nn}x_{n2} = 0 \end{cases} \qquad (2)$$

....

$$\begin{cases} a_{11}x_{1n} + a_{12}x_{2n} + \dots + a_{1n}x_{nn} = 1 \\ a_{21}x_{1n} + a_{22}x_{2n} + \dots + a_{2n}x_{nn} = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}x_{1n} + a_{n2}x_{2n} + \dots + a_{nn}x_{nn} = 0 \end{cases} \qquad (n)$$

2. Amaly paket programmalaryň algoritmik dillerinde ulanylyş aýratynlyklary.

Dürli görnüşli meseleleri programmirlemekde köplenç halatlarda birmeňzeş hasaplamalary şol programmada birnäçe gezek gaýtalap hasaplamak ýa-da dürli-dürli meselelerde hasaplamak üçin gerek bolup durýar. Şonuň ýaly ýagdaýlarda gaýtalanýan hasaplamalary aýratyn programma görnüşde düzmek (ýagny bölek programma görnüşde ýazmak) we gerek bolan ýerinde ony ulanmak (ýagny bölek programma ýüzlenmek) maksada laýyk bolup durýar. Programmirleme dilinde bölek programmanyň 2 sany görnüşi bar:

1) FUNCTION
2) SUBROUTINE

Bölek programmalara käbir bellikler bar:

1) Function görnüşli bölek programmany diňe netije bir üýtgeýän ululygynyň bahasy bolsa şol ýagdaýda ulanylýar.
2) Function görnüşli bölek programmada hökmany baha berme operatory bolmaly. Onda çep tarapda funksiýanyň ady bilen gabat gelýän üýtgeýän ululyk bolmaly. Sag tarapynda funksiýanyň alýan bahasy ýazylmaly.
3) Function görnüşli bölek programmada iň bolmanda bir Return operatory bolmaly.
4) Bölek programmanyň birinji setiri FUNCTION ýada SUBROUTINE görnüşli operator bilen başlanmaly we onda formal parametrleri kesgitlenmeli. Function görnüşli bölek programmada formal parametriň iň bolmanda birisiniň bolmagy hökmanydyr.
5) Eger formal parametrleriň deregine massiwler ulanylýan bolsalar, onda bölek programmada massiwleriň ölçegleri beýan edilmelidir.

6) Bölek programmanyň içinde FUNCTION we SUBROUTINE operatorlaryndan başga operatorlar gelip bilerler. Bölek programmanyň iň soňky operatory END bilen gutarmaly.
7) Bölek programma ýüzlenilen halatynda hakyky parametrleriň sany, görnüşi, tertibi hökmany gabat gelmelidir.
8) Function görnüşli bölek programmasyna ýüzlenmeklik diňe arifmetiki ýa-da logiki aňlatmalarda funksiýanyň adyndan soň hakyky parametrlerini ýazmak bilen amala aşyrylýar.
9) SUBROUTINE görnüşli bölek programmasyna CALL operatory bilen ýüzlenmeklik amala aşyrylýar.

Fortran dilinde funksiýa we bölek programma aşakdaky görnüşlerde ulanylýar.

a) Operator-funksiýa

$$a(b_1,b_2,...,b_n)=c$$

Bu ýerde: a- funksiýanyň ady

$b_1,b_2,...,b_n$- formal parametrler,

c- arifmetiki ýa-da logiki aňlatma

Goý programmanyň dürli ýerlerinde

$ax^2+by^2+cxy+d$

arifmetiki aňlatmanyň x we y dürli bahalarynda hasaplamak gerek bolsun. Onda operator-funksiýany f bilen atlandyryp aşakdaky görnüşde ýazmak bolar.

```
f(x,y)=a*x*2+b*y*y+c*x*y+d
```

bu operator funksiýa programmanyň başynda ýazylýar we programmada aşakdaky görnüşde ulanylýar.

```
y=x4+f(x1,y1)-f(x2,y2)
```

b) Bölekprogramma-funksiýa

Bölekprogramma-funksiýa operator funksiýadan tapawutlylykda birnäçe operator yzygiderligini ulanyp bilner. Ýagny bölekprogramma funksiýa özbaşdak programma bolup, ol başga programmalarda hem ulanylyp bilner. Bölekprogramma-funksiýanyň birinji operatory FUNCTION operatory bolup onuň umumy görnüşdäki ýazgysy aşakdaky ýalydyr.

t FUNCTION $f(a_1,a_2,...,a_n)$

Bu ýerde: f-bölekprogramma funksiýanyň ady,

$a_1,a_2,...,a_n$- formal parametrleriň sanawy (n>1)

t- operator belgisi ol ýazylman hem bilner.

c) Bölekprogrammanyň umumy görnüşi aşakdaky ýaly ýazylŷar

SUBROUTINE $a(b_1,b_2,...,b_n)$

Bu ýerde: a-bölekprogrammanyň ady,

$b_1,b_2,...,b_{n-}$ formal parametrler.

Bölekprogrammada iň bolmanda bir RETURN operatory bolmalydyr. Bölekprogramma END operatory bilen gutarmaly. Esasy programmada bölekprogramma aşakdaky ýaly görnüşde ýüzlenmeli.

CALL $a(c_1,c_2,...,c_n)$

Bu ýerde: a-ýüzlenilýän bölekprogrammanyň ady,

$b_1,b_2,...,b_{n-}$ parametrleriň hakyky bahalary.

CALL SIMQ (A,B,N,KS)

Bu ýerde: A- n*n ölçegli matrisa

B- n ölçegli massiw (azat agzasy) bölekprogramma ýerine ýetirilip bolmansoň x wektoryň hakyky bahalary.

N- deňlemeleriň sany.

KS-ýalňyşlyk kody.

2) Funksiýany interpolirleme (ALI)

CALL ALI (X,ARG,VAL,Y,ND,EPS,IER)

Bu ýerde: X- girizilýän x-iň bahasy, ýagny funksiýanyň bahasyny tapmaly nokat.

ARG- girizilýän wektor ululyk, ölçegli ND≤n

Val- girizilýän funksiýanyň bahalarynyň wektory, ölçegli ND≤n

Y-f(x) funksiýanyň hasaplanan bahasy.

ND- düwün nokatlaryň sany.

EPS- absolýut ýalňyşlygyň ýokary çägi (10^{-3} dan 10^{-6} çenli)

3) n ölçegli matrisa ýüzlenmek we matrisanyň kesgitleýjisini hasaplamak.

CALL MIN (A,N,D,L,M)

Bu ýerde: A-berlen matrisa, sütünler boýunça n*n bir ölçegli massiwde ýerleşdirilen, bölek programma ýerine ýetirilip bolansoň A^{-1}.

N- A matrisanyň tertibi.

D- A matrisanyň kesgitleýjisi.

L,M- işçi massiwler ölçegi N.

4) Matrisalary köpeltmek (GMPRD)

CALL GMPRD (A,B,C,N,M,L)

Bu ýerde: A-1-nji girizilýän matrisa.

B-2-nji girizilýän matrisa.

C- çykarylýan matrisa.

N- A matrisanyň setir sany.

M- A sütüni we B setiri.

L- B matrisanyň sütüni.

A,B,C- bir ölçegli massiwler ölçegleri degişlilikde N*M, M*L, N*L.

5) f(x)=0 deňlemäniň çözüwini hasaplamak (RTMI)

CALL RTMI (X,F,FCT,XL1,XRI,EPS,IEND,IER)

Bu ýerde: x-gözlenýän wektor ululyk

FCT(x)=0

F-kökde funksiýanyň bahasy, ýagny

F= FCT(x)

FCT- daşky Function görnüşli bölekprogramma, f(x)-iň bahasyny kesgitleýär.

XLI- kesimiň çep tarapy.

XRI- kesimiň sag tarapy.

EPS-takyklygy.

IEND- kesimi ikä bölmegiň maksimal bahasy.

IER-ýalňyş kody.

Mysal1. $2x-2x^2+\lg x-\dfrac{7}{2x+6}-1{,}5=0$

$E=10^{-4}$

```
   External FCC
   WRITE (3,15)
15 FORMAT ('...')
   CALL RTMI (X,F,FCC,6.0,7.0,1E-4,50,I)
   WRITE (3,10)x,y,i)
10 FORMAT ('x=', F8.4, 'F(x)=',F8.5,'I=',I3)
   STOP
   End
   FUNCTION FCC(x)
   FCC=2*x*2+ ALOC10(x)-7.1(2*x+6)-1.5
   RETURN
   END
```

Mysal2. $8x_1-x_2-2x_3=2.3$
$10x_1+x_2+2x_3=-0.5$
$-x_1+6x_2+2x_3=-1.2$
$3x_1-x_2+2x_3+12x_4=3.7$

```
      DIMENSION A(16),R1(4)
      WRITE (3,27)
   27 FORMAT ('...')
      READ (1,28) A,R1
   28 FORMAT (8F5.1)
      CALL SIMQ (A,R1,4,Ks)
      WRITE (3,25)R1
   25 FORMAT (T6,'kök='4F7.3)
      STOP
      END
```

Turbo paskalda funksiýalaryň we proceduralaryň ulanylşy.

Funksiýanyň umumy görnüşi

```
Function at (parametrleriň sanawy):kysym;
Begin
End;
```

Bu ýerde
At- harpdan başlanýan islendik simwollar yzygiderligi, parametrleriň sanawy aşakdaky görnüşleriň biri bolup biler.
at,at,...,at: kysym;
ýa-da at: kysym; at: kysym; at: kysym;
var at,at,...,at: kysym;
ýa-da var at: kysym; ...; at: kysym;
var- parametr-näbelli. Begin we end sözüniň içinde hökmany funksiýany bahalandyrýan operator bolmalydyr. Onuň aşakdaky ýaly görnüşi bar:

```
procedure at (parametrleriň sanawy);
```

```
Begin
End;
```

Ýöne parametrleriň sanawynyň içinde hökmany parametr näbelli bolmalydyr. Bu bolsa procedure we funksiýanyň esasy tapawutlarynyň biridir. Beýan edilen procedurany we funksiýany esasy programmada olaryň atlary hem-de hakyky parametrleri görkezmek arkaly amala aşyrylýar.

Biziň bilşimiz ýaly programma düzüji tarapyndan täze kysymlary döretmek we olaryň üstünde amallary kesgitlemek mümkinçiligi bardyr. Täze kysym programmanyň kysymlar bölüminde beýan edilmelidir. Her bir döredilýän kysymyň ady we onuň alyp biljek bahalary anyklanmalydyr. Täze döredilýän kysymy programmada type sözi bilen beýan edilmelidir. Programma düzüji tarapyndan döredilýän kysymda olaryň bahalaryny girizmek we çap etmek mümkinçiligi ýokdur. Şonuň üçin şeýle kysymly näbellileri bahalandyrmak CASE operatorynyň üsti bilen amala aşyrmak bolar. Bu operatoryň umumy görnüşi aşakdaky ýalydyr.

```
]       CASE  aňlatma operatory;
        baha 1: 1-nji operator;
        baha 2: 2-nji operator;
        baha n: n-nji operator;
      End;
```

Mysal. x,y hakyky san berlen. u=max(x+y, x*y), ν=max(0.5;4) hasaplamaly.

```
Program max;
var x,y,u,v,a,b,s:real;
procedure max2 (a,b:real; var s:real;)
begin
   if a>b then s:=a else s:=b end;
begin
   readln (x,y); a:=x+y; b:=x*y;
max2 (a,b,s); u:=s; a=0.5; b:=4;
```

```
max2 (a,b,s); v:=s; write(u,v);
end.
```

Amaly paket programmalaryň beýsik dilindäki aýratynlyklary.

1) Operator funksiýa onuň görnüşi.

DEF FN α(x)=E

Bu ýerde: DEF- funksiýany kesgitlemek üçin operator.
FN α - operator funksiýanyň ady bolup
FN hökmany ýazylmaly belgisi bolup durýar.
α – bolsa identifikator bolup programma düzüji tarapyndan kesgitlenilýär.
x- formal parametrleriň sanawy.
E- arifmetiki ýa-da logiki aňlatma bolup durýar.

Mysal. DEF FN f(x,y)= (X^2+Y^2)

Y=12.5+ FNf (-1,5)

b) Procedura funksiýa.

```
DEF FN α(x)
Operatorlar
FN END
```

Bu ýerde: FN α - procedura funksiýanyň ady.
x- formal parametrler “operatorlar”- beýsik dilindäki operatorlar.
FN END- procedura funksiýanyň soňy.

Procedura funksiýa ýüzlenmeklik FN α(A) ýazmaklyk bilen amala aşyrylýar.

2) BP (bölek programma)

Eger bölekprogrammada birnäçe operatorlar yzygiderligi gaýtalanyp gelýän bolsa, onda ony BP görnüşinde ulanmak amatly bolup durýar.

3. Amaly programmalar paketiniň çyzykly algebranyň meseleleri üçin düzülüşi.

Gaussyň usuly. Goỳ n näbellili n sany çyzykly algebraik deňlemeler ulgamy berlen bolsun

$$\begin{cases} a_{11}x_{11}+a_{12}x_2+\ldots+a_{1n}x_n=b_1, \\ a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n=b_2, \\ ------------------ \\ a_{n1}x_1+a_{n2}x_2+\ldots+a_{nn}x_n=b_n. \end{cases} \tag{1}$$

Goỳ, $a_{11} \neq 0$ (esasy element) diỳeliň we ulgamynyň birinji deňlemesiniň iki bőlegini hem a_{11} bőleliň. Netijede alarys

$$x_1+b_{12}x_2+\ldots+b_{1n}x_n=b_1^{(1)}, \tag{2}$$

bu ỳerde

$$b_{1j}=\frac{a_{1j}}{a_{11}},\ \ j=2,3,\ldots,n, b_1^{(1)}=\frac{b_1}{a_{11}}.$$

Berlen (1) ulgamyň ikinji deňlemesinden başlan x_1 agzaly goşulyjylary, (2) deňlemäniň kőmegi bilen ỳok edip alalyň. Onuň üçin (2) deňlemäni yzygiderlilikde $a_{21}, a_{31},...,a_{n1}$ kőpeldip we olary degişlilikde (1) ulgamyň ikinji, üçünji, …,n-nji deňlemelerden aỳryp alalyň. Netijede n-1 tertipli ulgamy alarys:

$$\begin{cases} a_{22}^{(1)}x_2 + \cdots + a_{2n}^{(1)}x_n = b_2^{(1)}, \\ -------------- \\ a_{n2}^{(1)}x_2 + \ldots + a_{nn}^{(1)}x_n = b_n^{(1)}, \end{cases}$$

bu ỳerde

$$a_{ij}^{(1)} = a_{ij} - a_{i1}a_{1j}, \quad i, j = 2,3,...,n,$$

$$b_i^{(1)} = b_i - a_i, b_1, \quad i = 2,3,...,n.$$

Ỳokardaky őzgertmeleri, alnan ulgam üçin hem ulanalyň. Netijede őzgertmeleri n – gezek gaytalap üçburçlyk günüşli matrisaly ulgamyny alarys

$$\begin{cases} x_1 + c_{11}x_2 + \ldots + c_{1n}x_n = d_1, \\ \qquad x_2 + \ldots + c_{2n}x_n = d_2, \\ -------------------- \\ \qquad\qquad x_n = d_n, \end{cases} \qquad (3)$$

Ýagny (1) ulgama ekwiwalent bolup we onuň çőzüwleri aňsat tapylỳär,iň soňky deňlemesinden x_n taparys, x_n-nyň bahasyny iň soňkynyň őň ỳanyndaky deňlemede goüp, x_{n-2} tapỳarys we ş.m

x_1 ulgamyň birinji deňlemesinden taparys. Şeỳlelikde Gausyň usulyny iki sany etapa bőlmek bolar, ỳagny berlen ulgamy ỳçburçlyk görnüşli matriza ulgamyna getirilişine gőni gidiş etapy, ỳçburçlyk gőrnüşli (3) ulgamyň çőzlülişine yza gidiş etapy dỳilỳär.Gausyň usulyny ulanmak üçin "esasy elementleriň" noldan tapawutly bolmagy hőhkmanydyr. Eger-de olaryň nola deňi bar bolsa,onda "esasy element" noldan tapawutly bolar ỳaly şol ulgamda deňlemeleriň ornuny üỳtgetmek gerekdir.

Gaussyň esasy elementi saỳlamak usuly. Çyzykly algebraik deňlemeler ulgamyna seredeliň. Onuň koeffisiỳentlerinden giňeldilen gőnüburçly görnüşli matrisany düzeliň.

$$M = \begin{pmatrix} a_{11}..a_{1q}..a_{1n} & b_1 \\ ------------ & \\ a_{p1}..a_{pq}..a_{pn} & b_p \\ ------------ & \\ a_{n1}..a_{nq}..a_{nn} & b_n \end{pmatrix} \qquad (4)$$

M – matrisanyň a_{ij} $(i, j = 1,2,\dots,n)$ elementleriň içinden moduly boýünça iň ulusyny saỳlalyň we ony "esasy element''diỳip alalyň.Goỳ ol a_{pq} element bolsun. Yagny ulgamyň $_p$ p-nji setirine onuň esasy setiri diỳilỳär. Onsoň m_i kőpagzany hasaplanyň

$$m_i = \frac{a_{iq}}{a_{pq}}$$

hemme $i \neq p$ üçin

Esasy seteri m_i kőpeldip (4) matrisada, i–nji esasy bolmadyk setirler bilen goşup alarys. Netijede q–nji sütünini we esasy setirini taşlap, biz täze M_1, ỳagny bir setiri we sütüni az bolan matrisany alarys.

M_1 matrisa üçin hem ỳokardaky operasiỳalary gaỳtalap, M_2 matrisany alarys we ş.m. Şunuň ỳaly operasiyalary, bir setirli (iki sany elementden durỳan) matrisa alnynỳança dowam edeliň. Şol bir setir esasy setir bolup durỳar. Hemme esasy setirleri birleşdirip we bir näçe üỳtgetmelerden soň (4) –e ekwiwalent bolan üçburuçlyk gőrnüşli matrisa emele geler. Şonuň bilen hasaplama etapy (gőni gidiş) tamamlanýar. Alnan üçburçlyk gőrnüşli matrisanyň koeffisiỳentlerinden düzülen ulgamy çőzüp, yzygiderli x_i - näbellileri tapỳarys. Bu hasaplamalara bolsa yzy gidiş etap diỳilỳär.

Kwadrat kőkler usuly. Goỳ,

$$A\vec{x} = \vec{b} \tag{5}$$

deňlemeler ulgamy berlen bolsun. Bu ỳerde A- kwadratik simetrik matrisa, $\vec{b}$ - azat agzaly wetor – sütün, $\vec{x}$ - näbellilerden düzülen wektor sütün. (5) ulgamyň çőzülüşini iki etaba bőleliň.

Gőnü gidiş etapy A matrisany iki sany üçburçlyk gőrnüşli transpornirilenen matrisallaryň kőpeltmek hasyly gőrnüşinde alalyň

$$A = T' \cdot T,$$

bu ỳerde

$$T = \begin{pmatrix} t_{11} t_{12} \dots t_{1n} \\ 0 t_{22} \dots t_{2n} \\ ------ \\ 0 0 \dots t_{nn} \end{pmatrix}$$

$$T' = \begin{pmatrix} t_{11} 0 \dots 0 \\ t_{12} t_{22} \dots 0 \\ t_{1n} t_{2n} \dots t_{nn} \end{pmatrix} \qquad (6)$$

T' bilen T kőpeldip we A matrisasy bilen deňeşdirip t_{ij} elementler üçin aşakdaky formulalary alarys.

$$t_{11} = \sqrt{a_{11},} \qquad t_{ij} \frac{a_{ij}}{t_{11}}, \qquad (j > 1),$$

$$t_{ii} = \sqrt{a_{ii} - \sum_{k=1}^{i-1} t_{ki}^2}, \qquad (1 < i = n),$$

$$t_{ij} = \frac{a_{ij} - \sum_{k=1}^{i-1} t_{ki} t_{kj}}{t_{ii}}, \qquad (i < j), \qquad (7)$$

$$t_{ij} = 0, \qquad (i > j).$$

T marisa tapylandan soň (5) ulgama ekwiwalent bolan, iki sany üçburçlyk gőrnỳşli matrisa ulgamyy bilen çalşalyň

$$T'\vec{y} = \vec{b}, \qquad T\vec{x} = \vec{y}.$$

(8)

Yza gidiş etapy. (8) ulgamy ỳaýraň gőrnüşinde ỳazalyň

$$\begin{cases} t_{11}y_1 = b_1, \\ t_{12}y_1 + t_{22}y_2 = b_2, \\ -------------------- \\ t_{1n}y_1 + t_{2n}y_2 + t_{nn}y_n = b_n \end{cases} . \quad (9)$$

$$\begin{cases} t_{11}x_1 + t_{12}x_2 + \ldots + t_{1n}x_n = y_1, \\ \qquad\qquad t_{22}y_1 + t_{2n}x_n = y_2, \\ -------------------- \\ \qquad\qquad\qquad t_{nn}x_n = y_n \end{cases} . \quad (10)$$

Bu ỳerden yzygiderlilikde taparys

$$y_1 = \frac{b_1}{t_{11}}, \quad y_i = \frac{b_i - \sum_{k=1}^{i-1} t_{ki} y_k}{t_{ii}}, \qquad (i > 1)$$

(11)

$$x_n = \frac{y_n}{t_{nn}}, \qquad x_i = \frac{y_i - \sum_{k=i+1}^{n} t_{ik} x_k}{t_{ii}}, \qquad (i < n).$$

(12)

Ỳőnekeỳ iterasiỳa usuly. Goỳ (1) çyzykly deňlemeler ulgamy aşakdaky gőrnüşe getirilen diýeliň

$$\vec{x} = C\vec{x} + \vec{f}, \tag{13}$$

bu ỳerde C-käbir matrisa, $\vec{f}$ -wektor – sütün .

Erkin $\vec{x}^{(0)}$ wektordan ugur alyp

$$\vec{x}^{(0)} = \begin{pmatrix} x_1^{(0)} \\ x_2^{(0)} \\ \dots\dots \\ x_n^{(0)} \end{pmatrix},$$

iterasion proses guralyň

$$\vec{x}^{(k+1)} = C\vec{x}^{(k)} + \vec{f}, \qquad (k = 0,1,2,...),$$

ỳa-da ỳaỳran gőrnüşinde

$$\begin{cases} x_1^{(k+1)} = c_{11}x_1^{(k)} + c_{12}x_2^{(k)} + ... + c_{1n}x_n^{(k)} + f_1, \\ ---------------------- \\ x_n^{(k+1)} = c_{n1}\ x_1^{(k)} + c_{n2}x_2^{(k)} + ... + c_{nn}x_n^{(k)} + f_n. \end{cases}$$

(14)

Iterasiỳany dowam etdirip $\vec{x}^{(1)}, \vec{x}^{(2)}, ..., \vec{x}^{(k)}$,... wektorlar yzygiderligini alarys.

Eger c matrisanyň elementleri aşakdaky şertleriň birini kanangatlandyrỳan bolsa.

$$\sum_{j=1}^{n} \left|C_{ij}\right| \le \alpha < 1, \qquad (i = 1,2.\dots,n),$$

$$\sum_{i=l}^{n} \left|C_{ij}\right| \le \beta < 1, \qquad (j = 1,2,\dots,n)$$

(15)

(16)

onda iterasiỳa prosesi, takyk $\vec{x}$ çozüwlerini ỳygnaỳar $\overrightarrow{x^{(0)}}$ wektoryň islendik başlangyç san bahalarynda. Şeỳlelik bilen, tükeniksis prosesden soň ulgamyň takyk çősüwi alynỳar. Islendik $\overrightarrow{x^{(0)}}$ wektor ysygidirligi, berlen ulgamyň ỳakynlaşan çőzüwleri bolup durỳär. Başlangyç $\overrightarrow{x^{(0)}}$ wektor erkin saýlanyp (kähalatlarda $\overrightarrow{x^{(0)}} = \overrightarrow{f^{(0)}}$) alynỳar. Berlen ulgamy (13) gőrnüşe,dürli-dürli usullar bilen getirmek bolar, ỳöne (15) ỳa-da (16) şertleriň iň bolmanda biriniň ỳerine etmegi hőkmandyr.

Zeỳdel usuly . Zeỳdeliň usuly ỳonekeỳ iterasiỳanyň modifikasiỳasy bolup durỳär. x_i $(i>1)$ näbellileriň $(k+1)$- nji bahalary tapylanda, $x_1, x_2, ..., x_{i-1}$ näbellilerin $(k+1)$-nji bahalary hem ulanylỳär. Şeỳlelik bilen.Zeỳdel usuly (13) ulgamy çőzmek üçin hasaplanylỳar

$$\begin{cases} x_1^{(k+1)} = c_{11}x_1^{(k)} + c_{12}x_2^{(k)} + \ldots + c_{1n}x_n^{(k)} + f_1 \,, \\ x_2^{(k+1)} = c_{21}x_1^{(k+1)} + c_{22}x_2^{(k)} + \ldots + c_{2n}^{(k)} f_2, \\ ------------------------ \\ x_n^{(k+1)} = c_{n1}x_1^{(k+1)} + c_{n2}x_2^{(k+1)} + \ldots + c_{n,n-1}x_{n-1}^{(k+1)} + c_{nn}x_n^{(k)} + f_n. \end{cases} \tag{17}$$

Zeỳdel usulynyň ỳygnanmagy üçin ỳőnekeỳ iterasiỳa şertleriniň ỳerine ỳetmegi hőkmanydyr. Kőplenç halatlarda Zeỳdel usuly ỳőnekeỳ iterasiỳa garanda çalt ỳygnanỳar. Zeỳdel usulyny programmirlemek, ỳőnekeỳ iterasiỳa garanda has amatlydyr, ỳagny $x_i^{(k+1)}$ tapmak üçin $x_1^{(k)}, \ldots, x_{i-1}^{(k)}$ bahalary saklamaklyk zerur bolup durmaỳar.

1 – nji mesele .

Berlen deňlemeler ulgamyny 0,0001 takyklykda Gaussyň kompakt we esasy elementi saỳlama shemalary boỳunça çőzmeli.

$$\begin{cases} 1{,}1161x_1 + 0{,}1254x_2 + 0{,}1397x_3 + 0{,}1490x_4 = 1{,}5471 \\ 0{,}1582x_1 + 1{,}1675x_2 + 0{,}1768x_3 + 0{,}1871x_4 = 1{,}6471 \\ 0{,}1968x_1 + 0{,}2071x_2 + 1{,}2168x_3 + 0{,}2271x_1 = 1{,}7471 \\ 0{,}2368x_1 + 0{,}2471x_2 + 0{,}2568x_3 + 1{,}2671x_1 = 1{,}8471 \end{cases}$$

Hemme hasaplamalaryň netijesini tablisa gőrnüşinde ỳazmaklyk amatlydyr. Gaussyň kompakt shemasy

i	a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{i5}	$\sum = a_{i6}$
1	1,1161	0,1254	0,1397	0,1490	1,5471	3,0773
2	0,1582	1,1675	0,1768	0,1871	1,6471	3,3367
3	0,1968	0,2071	1,2168	0,2271	1,7471	3,5949
4	0,2368	0,2471	0,2568	1,2671	1,8471	3,8549
	1	0,11235	0,12517	0,1335	1,38617	2,75719
2		1,14573	0,157	0,16598	1,42781	2,90051
3		0,18499	1,19217	0,20083	1,4743	3,05229
4		0,2205	0,22716	1,23549	1,51885	3,202
		1	0,13655	0,14436	1,24186	2,52277
3			1,16691	0,17412	1,24457	2,5856
4			0,19705	1,20366	1,24502	2,64573
			1	0,14921	1,06658	2,21579
4				1,17426	1,03486	2,20912
				1	0,88129	1,88129
			1		0,93505	1,93505
		1			0,98696	1,98696
	1				1,0406	2,0406

Näbellikleriň bahalaryny alarys

$$x_4 = 0,88129, \; x_3 = 0,93505, \quad x_2 = 0,98696, \; x_1 = 1,0406$$

.

Barlag üçin alnan ulgamyň çőzüwleri:

$$\overline{x_4} = 1,88129, \; \overline{x_3} = 1,93505, \quad \overline{x_2} = 1,98696, \; \overline{x_1} = 2,0406.$$

Alnan netijelerden $x_i + 1 = \overline{x_i}$ deňligiň dogrulygyny gőrmek bolỳar. Hasaplamanyň personal kompŷuterde alnan netijesi 1-nji programmada gőrkesilen.

```
cls
REM Gaussyn kompakt shemasy
dim a(4,4),b(4),x(4)
a(1,1)=1.1161:a(1,2)=0.1254:a(1,3)=0.1397:a(1,4)=0.1490
a(2,1)=0.1582:a(2,2)=1.1675:a(2,3)=0.1768:a(2,4)=0.1871
a(3,1)=0.1968:a(3,2)=0.2071:a(3,3)=1.2168:a(3,4)=0.2271
a(4,1)=0.2368:a(4,2)=0.2471:a(4,3)=0.2568:a(4,4)=1.2671
b(1)=1.5471:b(2)=1.6471:b(3)=1.7471:b(4)=1.8471
n=4
n1=n-1
for k=1 to n1
if a(k,k)<>0 then goto 4
i=k+1
6 if a(i,k)<>0 then goto 5
i=i+1
if i<=n then goto 6
print "DENlemeler ulgamynyn cozuwi yok":end
5 i=k
9 v=a(k,l):a(k,l)=a(i,l):a(i,l)=v
l=l+1
if l<=n then goto 9
v=b(k):b(k)=b(i):b(i)=v
```

```
4 j1=k+1
for j=j1 to n
a(k,j)=a(k,j)/a(k,k)
next j
b(k)=b(k)/a(k,k)
ki=k+1
for i=ki to n
for j=ki to n
a(i,j)=a(i,j)-a(i,k)*a(k,j)
next j
b(i)=b(i)-a(i,k)*b(k)
next i
next k
x(n)=b(n)/a(n,n)
k=n-1
26 r=0.0
j=n
23 r=r+a(k,j)*x(j)
if j-k>1 then j=j-1: goto 23
x(k)=b(k)-r
if k>1 then k=k-1: goto 26
print
print "Denlemeler ulgamynyn cozuwi:"
print
for i=1 to n
print "x("i")=";x(i)
next i
end
```

```
Denlemeler ulgamynyn cozuwi:

x( 1 )= 1.040583729743958
x( 2 )= .9869564771652222
x( 3 )= .9350525140762329
x( 4 )= .8812969923019409
```

1 – nji programma

Gausyň esasy element saỳlama shemasy

i	m_i	a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{i5}	$\sum a_{i6}$
1 2 3 4	0,1175 9 0,1476 6 0,1792 3	1,1161 0,1582 0,1968 0,2368	0,1254 0,1675 0,2071 0,2471	0,1397 0,1768 1,2168 0,2568	0,149 0,187 1 0,227 1 1,267 1	1,5471 1,6471 1,7471 1,8471	3,0773 3,3367 3,5949 3,8549
1 2 3	0,0935 3 0,1186 2	1,0882 5 0,1232 3 0,1543 6	0,0963 4 1,1310 1 0,1628 1	0,1095 0,1383 8 1,1707 7		1,3299 1,3743 6 1,4160 4	2,6239 9 2,7674 8 2,9039 8
1 2	0,0729 6	1,0738 1 0,1049 2	0,0811 1 1,1117			1,1974 6 1,2063 9	2,3523 8 2,4230 1
1		1,0661 6				1,1094 4	2,1756
1 2 3 4		1	 1	 1	 1	1,0405 9 0,9869 7 0,9350 5 0,8813	2,0405 9 1,9869 7 1,9350 5 1,8813

Hasaplamanyň personal kompŷuterde alnan netijesi 2-nji programmada gőrkezilen.

```
cls
REM Gaussyn sutunler boyunca esasy element saylama usuly
dim a(4,4),b(4),x(4)
a(1,1)=1.1161:a(1,2)=0.1254:a(1,3)=0.1397:a(1,4)=0.1490
a(2,1)=0.1582:a(2,2)=1.1675:a(2,3)=0.1768:a(2,4)=0.1871
a(3,1)=0.1968:a(3,2)=0.2071:a(3,3)=1.2168:a(3,4)=0.2271
a(4,1)=0.2368:a(4,2)=0.2471:a(4,3)=0.2568:a(4,4)=1.2671
b(1)=1.5471:b(2)=1.6471:b(3)=1.7471:b(4)=1.8471
n=4
n1=n-1
for k=1 to n1
i=k+1:m=k+1:l=k
203 if abs(a(m,k))>abs(a(l,k)) then l=m
if m<n then m=m+1: goto 203
if l=k then goto 208
i=k
210 v=a(l,i):a(l,i)=a(k,i):a(k,i)=v
if i<n then i=i+1: goto 210
v=b(k):b(k)=b(i):b(i)=v
208 c=a(i,k)/a(k,k):a(i,k)=0.0:j=k+1
1 a(i,j)=a(i,j)-c*a(k,j)
if j<n then j=j+1: goto 1
b(i)=b(i)-c*b(k)
if i<n then i=i+1: goto 208
next k
x(n)=b(n)/a(n,n)
k=n-1
26 r=0.0
j=n
23 r=r+a(k,j)*x(j)
if j-k>1 then j=j-1: goto 23
```

```
x(k)=(b(k)-r)/a(k,k)
if k>1 then k=k-1: goto 26
print
print "Denlemeler ulgamynyn cozuwi:"
print
for i=1 to n
print "x("i")=";x(i)
next i
end
```

Denlemeler ulgamynyn cozuwi:

x(1)= 1.040583848953247
x(2)= .9869564771652222
x(3)= .9350525140762329
x(4)= .8812969923019409

2 – nji programma

2-nji mesele

Berlen deňlemeler ulgamyny 0,001 takyklykda kwadrat kőkler usuly boỳunça çőzmeli.

$$\begin{cases} 4{,}25x_1 - 1{,}48x_2 + 0{,}73x_3 = 1{,}44 \\ -1{,}48x_1 + 1{,}73x_2 - 1{,}85x_3 = 2{,}73 \\ 0{,}73x_1 - 1{,}85x_2 + 1{,}93x_3 = -0{,}64 \end{cases}$$

T matrizanyň elementlerini tapmak üçin (7) formulada $n = 3$ diỳeliň

$$t_{11}=\sqrt{a_{11}}, \qquad t_{12}=\frac{a_{12}}{\sqrt{a_{11}}}, \qquad t_{13}=\frac{a_{13}}{\sqrt{a_{11}}};$$

$$t_{22}=\sqrt{a_{22}-t_{12}^2}, \qquad t_{23}=\frac{a_{23}-t_{12}t_{13}}{t_{22}};$$

$$t_{33}=\sqrt{a_{33}-t_{13}^2-t_{23}^2}.$$

Şu formulalaryň esasynda alarys

a_{i1}	a_{i2}	a_{i3}	a_{i4}	$\sum a_{i5}$
4,25 -1,48 0,73	-1,48 1,73 -1,85	0,73 -1,85 1,93	1,44 2,73 -0,64	4,94 1,13 0,17
2,0616	-0,7179 1,1021	0,3541 -1,448 0,5405	0,6985 2,9323 -6,2141	2,3962 2,5862 -5,6731
-2,020	-12,4446	-114969		
-1,0199	-11,4446	-10,4960		

Hasaplamanyň personal kompŷuterde alnan netijesi 3-nji programmada görkezilen.

```
cls
REM Kwadrat  kokler usuly
dim a(3,3),t(3,3),b(3),x(3),y(3)
a(1,1)=4.25:a(1,2)=-1.48:a(1,3)=0.73
a(2,1)=-1.48:a(2,2)=1.73:a(2,3)=-1.85
```

```
a(3,1)=0.73:a(3,2)=-1.85:a(3,3)=1.93
b(1)=1.44:b(2)=2.73:b(3)=-0.64
n=3
c=1
t(1,1)=sqr(a(1,1)):t(1,2)=a(1,2)/t(1,1):t(1,3)=a(1,3)/t(1,1)
t(2,1)=0.0
t(2,2)=sqr(a(2,2)-t(1,2)^2)
t(2,3)=(a(2,3)-t(1,2)*t(1,3))/t(2,2)
t(3,1)=0.0:t(3,2)=0.0
t3=a(3,3)-t(1,3)^2-t(2,3)^2
if t3<0 then t(3,3)=sqr(abs(t3)):c=-c: goto 104
t(3,3)=sqr(t3)
104 y(1)=b(1)/t(1,1)
y(2)=(b(2)-t(1,2)*y(1))/t(2,2)
y(3)=c*(b(3)-t(1,3)*y(1)-t(2,3)*y(2))/t(3,3)
x(n)=y(n)/t(n,n)
k=n-1
26 r=0.0
j=n
23 r=r+t(k,j)*x(j)
if j-k>1 then j=j-1: goto 23
x(k)=(y(k)-r)/t(k,k)
if k>1 then k=k-1: goto 26
print
print "Denlemeler ulgamynyn cozuwi:"
print
for i=1 to n
print "x("i")=";x(i)
next i
end
```

Denlemeler ulgamynyn cozuwi:

x(1)=-2.021391868591309
x(2)=-12.45080471038818

x(3)=-11.50174808502197

3 – nji programma

3-nji mesele.

Berlen deňlemeler ulgamyny 0,001 takyklykda ỳőnekeỳ iterasiỳa we Zeỳdel usullary boỳunca cőzmeli

$$\begin{cases} 20{,}9x_1 + 1{,}2x_2 + 2{,}1x_3 + 0{,}9x_4 = 21{,}7 \\ 1{,}2x_1 + 21{,}2x_2 + 1{,}5x_3 + 2{,}6x_4 = 27{,}46 \\ 2{,}1x_1 + 1{,}5x_2 + 19{,}8x_3 + 1{,}3x_4 = 28{,}76 \\ 0{,}9x_1 + 2{,}5x_2 + 1{,}3x_3 + 32{,}1x_4 = 49{,}72 \end{cases}$$

a) Berlen ulgamy aşakdaky gőrnüşde ỳazalyň

$$x_1 = (21{,}70 - 1{,}2x_2 - 2{,}1x_3 - 0{,}9x_4)/20{,}9$$

$$x_2 = (27{,}46 - 1{,}2x_1 - 1{,}5x_3 - 2{,}5x_4)/21{,}9$$

$$x_3 = (28{,}76 - 2{,}1x_1 - 1{,}5x_2 - 1{,}3x_4)/19{,}8$$

$$x_4 = (49{,}72 - 0{,}9x_1 - 2{,}5x_2 - 1{,}3x_4)/32{,}1$$

Alnan ulgamynyň koeffisiỳentleri (14) – şerti kanagatladyrỳär. Hakykatdan hem

$$\sum_{i=1}^{4}\left|c_{ij}\right| = 0,20 < 1, \qquad \sum_{i=1}^{4}\left|c_{ij}\right| = 0,24 < 1$$

$$\sum_{i=1}^{4}\left|c_{ij}\right| = 0,25 < 1 \qquad \sum_{i=1}^{4}\left|c_{ij}\right| = 0,15 < 1$$

Başlangyç $\overline{x^{(0)}}$ wektor üçin azat agzaly sütüni alalyň, ỳagny ol bahalary oturdan soň iki sifire çenli tegelekläp alarys:

$$\overline{x^{(0)}} = \begin{pmatrix} 1,04 \\ 1,30 \\ 1,45 \\ 1,55 \end{pmatrix}$$

Hasaplamalary $\left|x_i^k - x_i^{k-1}\right| \le 0,001$ şert ỳerine ỳetỳänçä dowam etdirmeli

k	x_1	x_2	x_3	x_4
0	1,04	1,30	1,45	1,55
1	0,75	0,95	1,14	1,36
2	0,8106	1,0118	1,2117	1,4077
3	0,7978	0,9977	1,1975	1,3983
4	0,8004	1,0005	1,2005	1,4003
5	0,7999	0,9999	1,1998	1|,3999

Hasaplamanyň personal kompŷuterde alnan netijesi 4 – nji programmada gőrkezilen.

cls
REM Yonekey iterasiya usuly
n=4

```
dim x(n),y(n)
y(1)=1.04:y(2)=1.30:y(3)=1.45:y(4)=1.55
j=0:eps=0.001
print " Denlemeler ulgamynyn cozuwi:"
print
print  " j      x(1)            x(2)            x(3)             x(4)"
print
10 print j;y(1);y(2);y(3);y(4)
for i=1 to n
x(i)=y(i)
next i
j=j+1
y(1)=(21.7-1.2*x(2)-2.1*x(3)-0.9*x(4))/20.9
y(2)=(27.46-1.2*x(1)-1.5*x(3)-2.5*x(4))/21.2
y(3)=(28.76-2.1*x(1)-1.5*x(2)-1.3*x(4))/19.8
y(4)=(49.72-0.9*x(1)-2.5*x(2)-1.3*x(3))/32.1
for i=1 to n
if abs(y(i)-x(i))>eps then goto 10
next i
end
```

```
Denlemeler ulgamynyn cozuwi:

 j      x(1)                  x(2)                  x(3)                 x(4)

0   1.03999996185    1.299999952316   1.450000047683
1.54999995231
1         .751196146011           .951037764549       1.14196968078
1.35978198051
2         .810373902320       1.01161110401            1.21152603626
1.4075317382
3         .7978509068489          .9977090954780      1.19752562046
1.398338079452
4        .8004516959190      1.000492691993         1.200510621070
1.400338888168
```

4 – nji programma

b) Hasaplamany Zeỳdel usuly boỳunça geçireliň

k	x_1	x_2	x_3	x_4
0	1,04	1,30	1,45	1,55
1	0,7512	0,9674	1,1977	1,4037
2	0,8019	1,9996	1,9996	1,4000
3	0,8001	0,0000	1,1999	1,4000

Hasaplamanyň personal kompŷuterde alnan netijesi 5 – nji programmada gőrkezilen.

```
cls
REM Zeydel usuly
n=4
dim x(n),y(n)
x(1)=1.04:x(2)=1.30:x(3)=1.45:x(4)=1.55
j=0:eps=0.001
print " Denlemeler ulgamynyn cozuwi:"
print
print  " j      x(1)             x(2)             x(3)              x(4)"
print
10 print j;x(1);x(2);x(3);x(4)
for i=1 to n
y(i)=x(i)
next i
j=j+1
x(1)=(21.7-1.2*x(2)-2.1*x(3)-0.9*x(4))/20.9
x(2)=(27.46-1.2*x(1)-1.5*x(3)-2.5*x(4))/21.2
x(3)=(28.76-2.1*x(1)-1.5*x(2)-1.3*x(4))/19.8
x(4)=(49.72-0.9*x(1)-2.5*x(2)-1.3*x(3))/32.1
```

```
for i=1 to n
if abs(y(i)-x(i))>eps then goto 10
next i
end
```

Denlemeler ulgamynyn cozuwi:

j	x(1)	x(2)	x(3)	x(4)
0	1.039999961853	1.299999952316	1.450000047683	1.549999952316
1	.7511961460113	.9673851132392	1.197798490524	1.403997540473
2	.8019216656684	.9995756149291	1.199565887451	1.399996757507
3	.8000681400299	1.000027298927	1.199990868568	1.399996280670

5 – nji programma

4. Amaly programmalar paketiniň interpolirleme meselesi üçin düzülüşi

Interpolirleme meselesiniň goỳuluşy. Goỳ $y = f(x)$ funksiỳasy tablisa gőrnüşde berlen bolsun:

$$y_0 = f(x_0), \quad y_1 = f(x_1), ..., y_n = f(x_n).$$

Interpoirleme meselesi adaty ỳagdaỳda aşakdaky gőrnüşde goỳulỳar: berlen x_i nokatlarda, degişlilikde $f(x)$ funksiỳalaryň bahalary bilen gabat gelỳän, derejesi n-den uly bolmadyk $p(x) = p_n(x)$ kőpagzany tapmaly.

Meseläniň geometrik manysy berlen $M_i(x_i; y_i)$ $(i = 0,1,2,...,n)$ nokatlar köplüginiň üstünden geçỳän.

$$y = a_0 x^n + a_1 x^{n-1} + ... + a_n$$

gőrnüşli algebraik ergini gurmaly. Interpolirleme meselesiniň ỳokardaky ỳaly goỳuluşyna, parabolic gőrnüşi mesele diỳilỳär, p(x) interpolirleleỳän funksiýa, $x_i (i = 0,1,2,...,n)$ nokatlara bolsa düwün nokatlary diỳilỳär.

Interpolision formulalar argumentiň aralyk bahalarynda näbelli $f(x)$- funksiỳasynyň bahalaryny kesgitlemekde ulanylỳar. Interpolirlemeklige iki hili garalỳar, ỳagny $x \in [x_0, x_n]$ ỳa-da $x \notin [x_0, x_n]$.

Lagranžyň interpolision formulasy. Goỳ, $x_i (i = 0,1,...,n)$- erkin düwün nokatlary, $y_i = f(x_i)$ funksiỳalary bolsa $f(x)$- funksiỳanyň bahalary diỳeliň . x_i nokatlarda y_i bahalary alỳan, n-derejeli kőpagza Lagranžyň interpolision formulasy diỳiilỳär.

$$L_n(x) = \sum_{i=0}^{n} y_i \frac{(x - x_0)(x - x_1)...(x - x_{i-1})(x_i - x_{i+1})...(x - x_n)}{(x_i - x_0)(x_i - x_1)...(x_i - x_{i-1})(x_i - x_{i+1})...(x_i - x_n)}. \tag{1}$$

Bu kőpagzanyň galyndy agzasy aşakdaky formula bilen kesgitlenỳär.

$$R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x - x_0)(x - x_i)...(x - x_n) \tag{2}$$

Bu ỳerde $\xi - f(x)$ *we* x_i düwün nokatlary saklaỳan iki kiçi aralygyň käbir nokady.

$$L_i^{(n)}(x) = \frac{(x-x_0)(x-x_1)...(x-x_{i-1})(x_i-x_{i+1})...(x-x_n)}{(x_i-x_0)(x_i-x_1)...(x_i-x_{i-1})(x_i-x_{i+1})...(x_i-x_n)}. \tag{3}$$

-aňlatma, Lagražnyň koeffisientleri diỳilỳär.

Lagranžyň koeffisientlerini hasaplamak üçin aşakdaky tapawutlaryň tablisiasyny peỳdalanmak amatlydyr (esasy diogonaldaky tapawutlaryň aşagy çyzylandyr).

$$\begin{array}{lllll}
 & \underline{x-x_0} & & & \\
 & x_1-x_0 & & & \\
 & x_2-x_0 & & & \\
x_0-x_1 & x_0-x_n & \dots & x_0-x_n & \\
\underline{x-x_1} & x_1-x_n & \dots & x_1-x_n & \\
x_2-x_1 & \underline{x-x_2} & \dots & x_2-x_n & \\
\end{array} \tag{4}$$

$$x_n-x_0 \quad x_n-x_1 \quad x_n-x_1 \quad \dots \quad \underline{x-x_n}$$

Tablisada i-nji setiriň elementleriniň kőpeltmek hasylyny D_i, diagonal elementleriniň kőpeltmek hasylyny bolasa $Ï_{n+1}(x)$

$$D_i = (x_i - x_0)(x_i - x_1)...(x_i - x_{i-1})(x - x_i)(x_i - x_{i+1})...(x_i - x_n),$$

$$\ddot{I}_{n+1}(x) = (x - x_0)(x - x_1)...(x - x_n),$$

$$L_i^{(n)}(x) = \frac{\ddot{I}_{n+1}(x)}{D_i}.$$

(5)

Eger $x = at + b,$ $x_j = at_j + b$ $(j = 0,1,...,n)$ bolsa, onda $L_i^{(n)}(x) = L_i^{(n)}(t).$ Deň aralykda düwün nokatlary üçin, Lagražyň koeffisiỳentleri berlendir, bu ỳagdaỳda hasaplama prosesi has ỳőnekeỳleşdirỳar.

Nỳutonyň interpolỳasion formulalary. Nỳutonyň interpolỳasion formulasy:

$$y(x) = p_n(x) = y_0 + q\Delta y_0 + \frac{q(q-1)}{2!}\Delta^2 y_0 + ... + \frac{q(q-1)...(q-n+1)}{n!}\Delta^n y_0$$

bu yerde $q = \frac{x - x_0}{h}, h = \frac{x_n - x_0}{n}.$

Bu formulada ỳokarky gorizontal setirdäki tükenikli tapawutlar ulanylyar. I-nji tablisada bu setiriň elementleriniň aşagy çyzyklandyr.

1-nji tablisa. Tükenikli tapawutlaryň gorizontal tablisasy

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	y_0	$\underline{\Delta y_0}$	$\underline{\Delta^2 y_0}$	$\underline{\Delta^3 y_0}$	$\underline{\Delta^4 y}_0$
x_1	y_1	Δy_1	$\Delta^2 y_1$	$\Delta^3 y_1$	$\Delta^4{}_1 y$
x_2	y_2	Δy_2	$\Delta^2 y_2$	$\Delta^3 y_2$	
x_3	y_3	Δy_3	$\Delta^2 y_3$		
x_4	y_4	$\Delta y4$			

x_5	y_5				

Bu ỳerde

$$\Delta y_i = y_{i+1} - y_i,$$

$$\Delta^2 y_i = \Delta y_{i+1} - \Delta y_i,$$

$$- - - - - - - - - - - - - -$$

$$\Delta^k y_i = \Delta^{k-1} y_{i+1} - \Delta^{k-1} y_i.$$

(6) – formulanyň galyndy agzasy $R_n(x)$ aşakdaky ỳaly kesgitlenỳär.

$$R_n(x) = h^{n+1} \frac{q(q-1)...(q-n)}{(n+1)!} f^{(n+1)}(\xi), \tag{7}$$

bu ỳerde ξ, x *we* x_i düwün nokatlary saklanỳan iň kiçi aralygyň käbir nokady.

Eger goşmaça x_{i+1} düwün nokady bolsa, onda praktikada has amatly takmyn formula ulanylỳar.

$$R_n(x) \approx \frac{\Delta^{n+1} y_0}{(n+1)!} q(q-1)...(q-n), \tag{8}$$

bu formula, funksiỳa empiric berlende has amatlydyr, n – sanyň $\Delta^n y_i$ hemişelik bolar ỳaly saỳlap almalydyr. Tablisada x nokada golaý interpolirlemek we ekstrapolirlemek üçin (6) formula ulanylỳar. n = I we n= 2 bolanda, (6) formuladan hususy ỳagdaỳlary alarys:

çyzykly interpolirleme

$$y(x) = y_0 + q\Delta y_0, \qquad (9)$$

kwdratik interpolirleme

$$y(x) = y_0 + q\Delta y_0 + \frac{q(q-1)}{2!}\Delta^2 y_0. \qquad (10)$$

Nỳutonyň ikinji interpolỳasion formulasy

$$y(x) = p_n(x) = y_n + q\Delta y_{n-1} + \frac{q(q+1)}{2!}\Delta^2 y_{n-2} + \ldots + \frac{q(q+1)\ldots(q+n-1)}{n!}\Delta^n y_0, \qquad (11)$$

bu ỳerde $q = \dfrac{x - x_n}{h}$.

Bu formulada aşaky ỳapgyt tükenikli tapawutlaryň setiri ulanylỳar. (11) – formulanyň galyndy agzasy $R_n(x)$ aşakdaky formula bilen kesgitlenỳär.

$$R_n(x) = h^{n+1}\frac{q(q+1)\ldots(q+n)}{(n+1)!} f^{(n+1)}(\xi), \qquad (12)$$

bu ỳerde ξ, x *we* x_i düwün nokatlary saklaỳan iň kiçi aralygyň käbir nokady.

Eger x tablisanyň soňyndaky x_n nokada golaỳ bolsa, onda (11) formula x nokada interpolirleme we ekstrapolirleme üçin ulanylỳar.

Gaussyň interpolỳasion formulalary. Gaussyň birinji (őňe interpolirleme üçin) interpolỳasion formulasy

$$p(x) = y_0 + q\Delta y_0 + \frac{q(q-1)}{2!}\Delta^2 y_{-1} + \frac{(q+1)q(q-1)}{3!}\Delta^3 y_{-1} + \frac{(q+1)q(q-1)(q-2)}{4!}\Delta^4 y_{-2} +$$

$$+\frac{(q+2)(q+1)q(q-1)(q-2)}{5!}\Delta^5 y_{-2} + \ldots + \frac{(q+n-1)\ldots(q-n+1)}{(2n-1)!}\Delta^{2n-1} y_{-(n-1)} +$$

$$+\frac{(q+n-1)\ldots(q-n)}{(2n)!}\Delta^{2n} y_{-n}, \qquad (13)$$

bu ỳerde $q = \dfrac{x - x_0}{h}$.

(13) – formuladaky $\Delta y, \Delta^2 y_{-1}, \Delta^3 y_{-1}, \Delta^4 y_{-2}, \Delta^5 y_{-2}, \ldots$ tapawutlar 2 – nji tablisadan aşakdaky düwük çyzyk boỳunça alnandyr.

Gaussyň ikinji (iza interpolirleme üçin) interpolỳasion formulasy

$$p(x) = y_0 + q\Delta y_{-1} + \frac{q(q+1)}{2!}\Delta^2 y_{-1} + \frac{(q+1)q(q-1)}{3!}\Delta^3 y_{-2} + \frac{(q+2)(q+1)q(q-1)}{4!}\Delta^4 y_{-2} +$$

$$\ldots + \frac{(q+n-1)\ldots(q-n+1)}{(2n-1)!}\Delta^{2n-1} y_{-n} + \frac{(q+n)(q+n-1)\ldots(q-n+1)}{(2n)!}\Delta^{2n-1} y_{-n} \qquad (14)$$

bu ỳerde $q = \dfrac{x - x_0}{h}$.

(14)- formuladaky $\Delta y_{-1}, \Delta^2 y_{-1}, \Delta^3 y_{-2}, \Delta^4 y_{-2}, \Delta^5 y_{-3}, \ldots$ tapawutlar 2-nji tablisadadan ỳokarky dőwük çyzyk boỳunça alanadyr. (13) we (14) formulalaryň galyndy agzalary aşakdaky formula bilen kesgitlernỳär.

$$R_n(x) = \frac{h^{2n+1} f^{(2n+1)}(\xi)}{(2n+1)!} q(q^2 - 1^2)(q^2 - 2^2)...(q^2 - n^2),$$

bu ỳerde ξ, x *we* x_i düwün nokatlary saklaỳan iň kiçi aralygyň käbir nokady.

2-nji tablisa. Tapawutlaryň diagonal tablisasy

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
x_{-4}	y_{-4}						
x_{-3}	y_{-3}	Δy_{-4}	$\Delta^2 y_{-4}$				
x_{-2}	y_{-2}	Δy_{-3}	$\Delta^2 y_{-3}$				
x_{-1}	y_{-1}	Δy_{-2}	$\Delta^2 y_{-2}$	$\Delta^3 y_{-4}$			$\Delta^6 y_{-4}$
x_0	y_0	Δy_{-1}	$\Delta^2 y_{-1}$	$\Delta^3 y_{-3}$	$\Delta^4 y_{-4}$		$\Delta^6 y_{-3}$
x_1	y_1	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_{-2}$	$\Delta^4 y_{-3}$	$\Delta^5 y_{-4}$	$\Delta^6 y_{-2}$
x_2	y_2	Δy_1	$\Delta^2 y_1$	$\Delta^3 y_{-1}$	$\Delta^4 y_{-2}$	$\Delta^5 y_{-3}$	
x_3	y_3	Δy_2	$\Delta^2 y_2$	$\Delta^3 y_0$	$\Delta^4 y_{-1}$	$\Delta^5 y_{-2}$	
x_4	y_4	Δy_3		$\Delta^3 y_1$	$\Delta^4 y_0$	$\Delta^5 y_{-1}$	

Eger x tablisanyň ortasynda x_0 nokada golaỳ bolsa, anda Gaussyň formulalaryny ulanmak, ỳagny $x > x_0$ bolsa (13) formula, $x < x_0$ bola, (14) formula amatlydyr.

Stirlingiň interpolỳasion formulasy.

$$P(x) = y_0 + q\frac{\Delta y_{-1} + \Delta y_0}{2} + \frac{q^2}{2}\Delta^2 y_{-1} + \frac{q(q^2-1^2)}{3!}\frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2} + \frac{q^2(q^2-1^2)}{4!}\Delta^4 y_{-2} +$$

$$+\frac{q(q^2-1^2)(q^2-2^2)}{5!}\frac{\Delta^5 y_{-3} + \Delta^5 y_{-2}}{2} + \frac{q^2(q^2-1^2)(q^2-2^2)}{6!}\Delta^6 y_{-3} + \ldots + \frac{q(q^2-1^2)\ldots[q^2-(n-1)^2]}{(2n-1)!} *$$

$$\frac{\Delta^{2n-1} y_{-n} + \Delta^{2n-1} y_{(n-1)}}{2} + \frac{q^2(q^2-1^2)\ldots[q^2-(n-1)^2]}{(2n)!}\Delta^{2n} y_{-n}, \tag{16}$$

bu ỳerde $q = \dfrac{x - x_0}{h}$.

(16) – formulanyň galyndy agzasy hem (15) – formula bilen kesgitlenỳär.

Interpolirleme üçin Stirlingiň formulasy tablisanyň ortasynda, q–yň nola golaỳ bahalarynda ulanmak amatlydyr. Praktikada $|q| \le 0{,}25$ bahalarynda ulanylỳar.

Besseliň interpolỳasion formulasy.

$$P(x)=\frac{y_0+y_1}{2}+\left(q-\frac{1}{2}\right)\Delta y_0+\frac{q(q-1)}{2}\frac{\Delta^2 y_{-1}+\Delta^2 y_0}{2}+$$

$$+\frac{(q-0,5)q(q-1)}{3!}\Delta^3 y_{-1}+\frac{q(q-1)(q+1)(q-2)}{4!}\frac{\Delta^4 y_{-2}+\Delta^4 y_{-1}}{2}+\frac{(q-0,5)q(q+1)(q-2)}{5!}\Delta^5 y_{-2}+$$

$$+\frac{q(q-1)(q+1)(q-2)(q+2)(q-3)}{6!}\frac{\Delta^6 y_{-3}+\Delta^6 y_{-2}}{2}+\ldots+\frac{q(q-1)(q+1)(q-2)(q+2)\ldots(q-n)(q+n-1)}{(2n)!}$$

$$\frac{\Delta^{2n} y_{-n}+\Delta^{2n} y_{-n+1}}{2}+\frac{(q-0,5)q(q-1)(q+1)(q-2)(q+2)\ldots(q-n)(q+n-1)}{(2n+1)!}\Delta^{2n+1} y_{-n}, \qquad (17)$$

bu ỳerde $q=\dfrac{x-x_0}{h}$.

(17) – formulanyň galyndy agzasy aşakdaky formula bilen kesgitlenỳär.

$$R_n(x)=\frac{h^{2n+1}}{(2n+2)!}f^{(2n+2)}(\xi)q(q^2-1^2)\ldots(q^2-n^2)(q-n-1),$$

bu ỳerde ξ, x_0-nh *we* x_0+nh düwün nokatlarynyň araligynda ỳatỳan içki nokat.

Interpolirleme üçin Besseliň formulasy tablisanyň ortasynda q-yň 0,5-e golaỳ bahalarynda ulanmak amatlydyr. Praktikada $0,25\le q\le 0,75$ bahalarynda ulanylỳar.

q=0,5 üçin galyndy agza $R_n(x)$ aşakdaky formula bilen kesgitlenỳär.

$$R_n(x)=\frac{(-1)^{n+1}h^{2n+2}}{(2n+2)!}f^{(2n+2)}(\xi)\frac{[1\cdot 2\cdot 3\cdot\ldots\cdot(2n+1)]^2}{2^{2n+2}}$$

1-nji mesele.

Lagranžyň interpolision kőpagzasynyň kőmegi bilen, berlen düwün nokatlaryndaky bahalary ulanyp, $f(x)$ - funksiỳanyň berlen x nokatdaky takmyn bahasyny tapmaly: a) Eger düwün nokatlary deň däl aralyk gőrnüşinde berlen bolsa, (3-nji tablisa): b) Eger düwün nokatlary deň aralyk gőrnüşinde berlen bolsa, (4-nji tablisa).

3-nji tablisa

a)

x	y
0,05	0,050042
0,10	0,100335
0, 17	0, 171657
0,25	0, 255342
0,30	0, 309336
0,36	0, 376403

$x=0{,}263$ bahasynda $f(x)-$ funksiyanyň takmyn bahasyny hasaplamaly .

4-nji tablisa

b)

x	y
0,101	1,26183
0,106	1,27644
0,111	1,29122
0,116	1,30617
0,121	1,32130
0,126	1,32660

$x=0{,}1157$ bahasynda $f(x)-$ funksiyanyň takmyn bahasyny hasaplamaly.

a) Hasaplama aşakdaky formula bilen geçirilỳär.

$$f(x) \approx \Pi_{n+1} \sum_{i=0}^{n} (y_i / D_i),$$

bu ỳerde

$$\Pi_{n+1} = (x - x_0)(x - x_1)\ldots(x - x_n),$$

$$D_i = (x_i - x_0)\ldots(x_i - x_{i-1})(x_i - x_{i+1})\ldots(x_i - x_n).$$

Hasaplamalaryň netijeleri aşakdaky tablisada ỳerleşdirilendir.

i	Tapawutlar						D_i	y_i/D_i
0	0,213	-0,05	-0,12	-0,20	-0,25	-0,31	$-0,19809*10^{-4}$	$-2526,2$
1	0,05	0,163	-0,07	-0,15	-0,20	-0,26	$0,44499*10^{-5}$	$25547,7$
2	0,12	0,07	-0,093	-0,08	-0,13	-0,19	$-0,154365*10^{-5}$	$-111202,0$
3	0,20	0,15	0,08	0,013	-0,05	-0,11	$0,1716*10^{-6}$	$1488007,0$
4	0,25	0,20	0,13	0,05	-0,037	-0,06	$0,7215*10^{-6}$	$428740,0$
5	0,31	0,26	0,19	0,11	0,06	-0,097	$-0,980402*10^{-6}$	$-38392,7$

Diỳmek. $\Pi_{5+1} = 0.1506492 \cdot 10^{-6}$, $\sum_{i=0}^{5} (y_i / D_i) = 1790173,8.$

Onda,

$$f(0,263) = \Pi_{5+1} \cdot \sum_{i=0}^{5} (y_i / D_i) = 0,1506492 \cdot 10^{-6} \cdot 1790173,8 = 0,269678.$$

Hasaplamanyň personal kompŷuterde alnan netije 1-nji programmada gőrkezilendir.

cls

REM Lagranjyn den dal aralyk ucin formulasy

```
dim x(n),y(n)
input "Duwun nokatlaryn sanyny giriz n=",n
print
print "Duwun nokatlaryn we funksiyanyn bahalaryny giriz"
print
for i=0 to n
input x(i),y(i)
next i
input "Argumentin san bahasyny giriz z=",z
g=1.0
for j=0 to n
g=g*(z-x(j))
next j
s=0.0
for j=0 to n
h=g/(z-x(j))
d=1.0
for i=0 to n
if i=j then goto 13
d=d*(x(j)-x(i))
13 next i
s=s+y(j)*h/d
next j
print
print "Meselanin jogaby:"
print
print "y("z")=",s
end
```

```
Duwun nokatlaryn sanyny giriz n=5

Duwun nokatlaryn we funksiyanyn bahalaryny giriz

? 0.05,0,050042
```

? 0.10,0.100335
? 0.17,0.171657
? 0.25,0.255342
? 0.30,0.309336
? 0.36,0.376403

Argumentin san bahasyny giriz z=0.263

Meselanin jogaby:

y(.2630000114440918)=   .2696170210838318

1 – nji programma

b) Hasaplama aşakdaky formula bilen gőrkezilỳär.

$$f(x) \approx \Pi_{n+1}(t)\sum_{i=0}^{n}\frac{y_i}{(t-i)c_i},$$

bu ỳerde

$$\Pi_{n+1}(t) = t(t-1)...(t-n),$$

$$t = (x-x_0)!h;\ h = x_{i+1}-x_i;\ C_i = (-1)^{n-i}\cdot i!\cdot(n-i)!$$

Hasaplamalaryň netijeleri aşakdaky tablisada ỳerleşdirlendir.

i	x_i	y_i	$t-i$	c_i	$(t-i)c_i$	$\frac{y_i}{(t-i)c_i}$
0	0,101	1,26183	2,94	-120	-352,8	-0,0035766
1	0,106	1,27644	1,94	24	46,56	0,0274149

2	0,111	1,29122	0,94	-12	-11,28	-0,1144691
3	0,116	1,30617	-0,06	12	-0,72	-1,8141250
4	0,121	1,32130	-1,06	-24	25,44	0,0519379
5	0,126	1,33660	-2,06	120	-247,2	-0,0054069

Diỳmek,

$$\Pi_{5+1}(t) = 0{,}7024271; \qquad \sum_{i=0}^{5} \frac{y_i}{(t-i)c_i} = -1{,}858223.$$

Onda,

$$f(0{,}1157) \approx (-1{,}7024271)(-1{,}858225) = 1{,}30527.$$

Hasaplamanyň personal kompŷuterde alnan netije 2-nji programmada gőkezilendir.

```
cls
rem Lagranjyn den aralyk formulasy
dim x(5),y(5),c(5)
x(0)=0.101:x(1)=0.106:x(2)=0.111
x(3)=0.116:x(4)=0.121:x(5)=0.126
y(0)=1.26183:y(1)=1.27644:y(2)=1.29122
y(3)=1.30617:y(4)=1.32130:y(5)=1.32660
c(0)=-120:c(1)=24:c(2)=-12:c(3)=12
c(4)=-24:c(5)=120
z=0.1157
n=5:h=0.005
t=(z-x(0))/h
p=1:s=0
for i=0 to n
p=p*(t-i)
s=s+y(i)/((t-i)*c(i))
next i
s=s*p
print
print "Meselanin jogaby:"
print
print "y("z")=",s
```

end

Meselanin jogaby:

y(.1156999990344048)=   1.305239677429199

2 – nji programma

2 – nji mesele

Nỳutonyň birinji ỳa – da ikinji interpolỳasion formulalarynyň kőmegi bilen bilen berlen düwün nokatlaryndaky bahalary ulanyp (5 – nji tablisa) $f(x)$ - funksiỳanyň berlen $x_1 \, we \, x_2$ nokatlardaky takmyn bahasyny tapmaly (6-njy tablisa). Tapawut tablisasy düzülende barlag hasaplamalaryny geçirmeli.

x	y

1,215	0,106044
1,220	0,106491
1,225	0,106935
1,230	0,107377
1,235	0,107818
1,240	0,108257
1,245	0,108696
1,250	0,109134
1,255	0,109571
1,260	0,110008

Argumentiň $x_1 = 1,2173$ *we* $x_2 = 1,270$ bahalarynda $f(x)$ – funksiỳanyň takmyn bahalaryny hasaplamaly. Tükenikli tapawutlaryň tablisasyny düzeliň. Barlag hasaplamalaryny geçirmek üçin oňa iki setir goşalyň: $\sum$ bilen bellenen setirde Δy_i *we* $\Delta^2 y_i$ sütünlerdäki elementleriň jemini, P bilen bellenen setirde bolsa, y_i *we* Δy_i sütünlerdäki çetki elementleriň tapawudyny ỳazarys. Tükenikli tapawutlary hasaplamyzda 2 – nji tertipli tapawutlar bilen çäklendiris.

i	x_i	y_i	Δy_i	$\Delta^2 y_i$
1	1,215	0,106044	0,000447	-0,000003
2	1,220	0,106491	0,000444	-0,000002
3	1,225	0,106935	0,000442	-0,000001
4	1,230	0,107377	0,000441	0,0
5	1,235	0,107818	0,000439	-0,000001
6	1,240	0,108257	0,000438	-0,000001
7	1,245	0,108696	0,000437	0,0
8	1,250	0,109134	0,000437	-
9	1,255	0,109571	-	-
10	1,260	0,110008		
$\sum$	-	-	0,003964	-0,000010

P	-	0,003964	-0,000010	-

Eger $x = 1{,}2173$ bolsa, onda $q = (1{,}21173 - 1{,}215)/0{,}005 = 0{,}46$ (6)-formulany ulanyp, alarys

$$f(1{,}2173) \approx 0{,}106044 + 0{,}46 \cdot 0{,}000447 + \frac{0{,}46(-0{,}54)}{2} 2(-0{,}000003) = 0{,}106250.$$

Eger $x = 1{,}270$ bolsa, onda $q = (1{,}270 - 1{,}260)/0{,}005 = 2$ (II)- formulany ulanyp, alarys.

$$f(1{,}270) \approx 0{,}110008 + 2 \cdot 0{,}000437 + \frac{2 \cdot 3}{2}(-0{,}000001) = 0{,}110879.$$

Hasaplamanyň personal kompŷuterde alnan netijeleri 3-nji programmada gőŕkezilendir.

```
cls
REM Nyutonyn den aralyk ucin formulalary
dim x(11),y(11), Dy1(11), Dy2(11)
N=10: x1=1.2173: x2=1.270
x(1)=1.215:x(2)=1.220:x(3)=1.225:x(4)=1.230:x(5)=1.235:x(6)=1.2
40
x(7)=1.245:x(8)=1.250:x(9)=1.255: x(10)=1.260
y(1)=.106044:y(2)=.106491:y(3)=.106935:y(4)=.107277:y(5)=.1078
18
y(6)=.108257:y(7)=.108696:y(8)=.109134:y(9)=.109571:y(10)=.110
008
for i=1 to 9
Dy1(i)=y(i+1)-y(i)
next i
for i=1 to 8
Dy2(i)=Dy1(i+1)-Dy1(i)
```

```
next i
h=(x(n)-x(1))/n
q(1)=(x1-x(1))/h
q(2)=(x2-x(n))/h
f(1)=y(1)+q(1)*Dy1(1)+(q(1)*(q(1)-1)*Dy2(1))/2
f(2)=y(n)+q(2)*Dy1(n-1)+(q(2)*(q(2)+1)*Dy2(n-2))/2
print
print "Meselanin jogaby:    "
print "    "
print "x1=1.2173 bolanda f(1)=",f(1)
print "x2=1.270 bolanda f(2)=",f(2)
end
```

```
Meselanin jogaby:

x1=1.2173 bolanda f(1)=      .1062728464603424
x2=1.270 bolanda f(2)=       .1109791100025177
```

3 – nji programma

3-nji mesele

Gaussyň, Stirlingiň we Besseliň interpolỳasion formulalarynyň kőmegi bilen, berlen düwün nokatlaryndaky bahalary ulanyp $f(x)$- funksiỳanyň berlen x nokatdaky takmyn bahasyny tapmaly.

x	$y(x)$

1,50	15,132
1,55	17,422
1,68	20,393
1,65	23,994
1,70	28,160
1,75	32,812
1,80	37,857
1,85	43,189
1,90	48,699
1,95	54,225
2,00	59,653
2,05	64,817
2,10	69,550

3-nji meseläniň birisiniň çőzülişi.

$a)\ x = 0{,}163$

$b)\ x = 0{,}192$

$c)\ x = 0{,}204$

$d)\ x = 0{,}175$

bahalarynda $f(x)$ - funksiỳanyň takmyn bahalaryny hasaplamaly. Tapawutlaryň diagonal tablisasyny düzeliň.

x	$y(x)$
0,12	6,278
0,14	6,404
0,16	6,487
0,18	6,505
0,20	6,436
0,22	6,259
0,24	6,954

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$
0,12	6,278	0,126		
0,14	6,404	0,083	-0,043	-0,022
0,16	6,487	0,018	-0,065	-0,022
0,18	6,505	0,069	-0,087	-0,021
0,20	6,436	0,177	-0,108	-0,020
0,22	6,259	-0,305	-0,128	
0,24	5954			

Tapawutlar hasaplananda 3 – nji tertipli tapawutlar bilen çäklenendiris.

a) y(0,168) bahasyny hasaplamak üçin x_0=0,16 diỳeliň, onda $q=(x-x_0)/h$=(0,168-0,16)/0,02=0,4

(13)-formulany ulanyp alarys

$$y(0{,}168)\approx 6{,}487+0{,}4\cdot 818-\frac{0{,}4(-0{,}6)}{2}(-0{,}065)+\frac{1{,}4\cdot 0{,}4\cdot(-0{,}6)}{6}(-0{,}022)\approx 6{,}503.$$

b) y(0,192) bahasyny hasaplamak üçin x_0==0,18 diỳeliň, anda

q=(0,192-0,18)/0,02=0,6

(17)- formulany ulanyp, alarys

$$y(0{,}192)\approx\frac{6{,}505+6{,}436}{2}+(0{,}6-0{,}5)(-0{,}069)+\frac{0{,}6+0{,}4}{2}\,\frac{-0{,}087-0{,}108}{2}\;\frac{(0{,}6-0{,}5)\cdot 0{,}6(-0{,}4)}{6}(-0{,}021)\approx 6{,}475$$

c) y(0,204) bahasyny hasaplamak üçin x_0 =0,20 diỳeliň onda
q=(0,204-0,20)/0,02=0,2
(16)- formulany ulanyp, alarys

$$y(0{,}204)\approx 6{,}436+\frac{-0{,}069-0{,}177)}{2}0{,}2+\frac{0{,}04}{2}(-0{,}108)+\frac{0{,}2(0{,}04-1)}{6}\cdot\frac{-0{,}021-0{,}020}{2}\approx 6{,}410.$$

d) y(0,175) bahasyny hasaplamak üçin x_0 ==0,18 diỳeliň, anda
q=(0,175-0,18)/0,02=-0,25
(14)-formulany ulany, alary

$$y(0{,}175)\approx 6{,}505+(-0{,}25)\cdot 0{,}018+\frac{0{,}75-0{,}25}{2}(-0{,}087)+\frac{0{,}75(-0{,}25)(-1{,}25)}{6}(-0{,}022)\approx 6{,}508.$$

Hasaplamalaryň personal kompŷuterde alnan netijeleri 4-nji programmada gőrkezilendir.

```
cls
REM Gayss,Stirling, Bessel den aralyk ucin formulalary
dim x(11),y(11), y1(11), y2(11),y3(11)
t(1)=0.163: t(2)=0.192:t(3)=0.204:t(4)=0.175
x0(1)=0.16:x0(2)=0.18:x0(3)=0.20:x0(4)=0.18
x(1)=0.12:x(2)=0.14:x(3)=0.16:x(4)=0.18:x(5)=0.20:x(6)=0.22
x(7)=0.24
y(1)=6.278:y(2)=6.404:y(3)=6.487:y(4)=6.505:y(5)=6.436
y(6)=6.259:y(7)=5.954
for i=1 to 7
y1(i)=0
y2(i)=0
y3(i)=0
```

```
next i
for i=1 to 6
y1(i)=y(i+1)-y(i)
next i
for i=1 to 5
y2(i)=y1(i+1)-y1(i)
next i
for i=1 to 4
y3(i)=y2(i+1)-y2(i)
next i
h=(x(7)-x(1))/7
for i=1 to 4
q(i)=(t(i)-x0(i))/h
next i
f(1)=y(3)+q(1)*y1(3)+q(1)*(q(1)-1)*y2(3)/2+(q(1)+1)*q(1)*(q(1)-
1)*y3(3)/6
f(2)=(y(4)+y(3))/2+(q(2)-0.5)*y1(4)+q(2)*(q(2)-1)*(y2(3)+y2(4))/4
f(2)=f(2)+(q(2)-0.5)*q(2)*(q(2)-1)*y3(3)/6
f(3)=y(5)+q(3)*(y1(4)+y1(5))/2+q(3)^2*y2(4)/2+q(3)*(q(3)^2-
1)*(y3(3)+y3(4))/12
f(4)=y(4)+q(4)*y1(3)+q(4)*(q(4)+1)*y2(3)/2+(q(4)+1)*q(4)*(q(4)-
1)*y3(2)/6
print
print "Meselanin jogaby:    "
print
for i=1 to 4
print "x=";t(i);" bolanda f = ";f(i)
next i
end
```

Meselanin jogaby:

x= .1630000025033951 bolanda f = 6.497024059295654
x= .1920000016689301 bolanda f = 6.492584705352783

x= .2039999961853027 bolanda f = 6.405113697052002
x= .1749999970197678 bolanda f = 6.507758617401123

4 – nji programma

5. Amaly programmalar paketiniň kesgitlenen integraly çözmeklik üçin düzülüşi

Matematika derňew dersinden belli bolşy ỳaly, eger $f(x)$- funksiỳa $[a,b]$ kesimde üznüksiz bolsa, onda bu funksiỳadan a-dan b-e çenli alnan integral aşakdaky ỳaly hasaplanỳar

$$\int_a^b f(x)dx = F(b) - F(a),$$

bu ỳerde $F(x)$- $f(x)$ funksiỳanyň asyl funksiỳasydyr.

Emma Nỳuton-Leỳbnisiň formulasyndan peỳdalanyp bolmaỳan wagtlary hem bardyr. Eger berlen integral aşagyndaky $f(x)$ funksiỳanyň $F(x)$ asyl funksiỳasyny tapmak kyn bolsa ỳa-da $f(x)$ funksiỳa üçin $F(x)$ asyl funksiỳa düỳbünden ỳok bolsa, şeỳle hem $f(x)$ funksiỳa tablisa gőrnüşinde berlen bolsa, onda Nỳuton-Leỳbnisiň formulasyny ulanyp bolmaýar. Bu ýagdaýlarda kesgitlenen integralyň takmyn bahasyny tapmak formulalaryny peỳdalanmak amatlydyr. Takmyn integrirleme formulalaryna kwadratura formulalar diỳilỳär. Integral aşakdaky funksiỳany haỳsy hem bolsa bir interpolỳasion köpagza bilen çalşyryp kwadratyr formulasy alynỳar.

$$\int_a^b f(x)dx = \sum_{k=0}^{n} A_k f(x_k) + R.$$

(1)

bu ỳerde, x saỳlanyp alnan düwün nokady; A_k (k=0,1,…,n) $f(x)$ funksiỳa bagly bolmadyk, diňe düwün nokadynyň saỳlanyp alnyşyna bagly bolan koeffisientler.

R – galyndy agza ya-da kwadratyr formulanyň hatasy.

Nỳuton – Kotesiň formulasy.

$$\int_a^b f(x)dx = (b-a)\sum_{i=0}^{n} H_i y_i,$$

(2)

bu ỳerde H_i (i=0,1,…,n) – Nỳuton – Kotesiň koeffisientleri, ol n – e bagly bolup , $f(x)$ funksiỳa bagly bolmadyk sandyr. Käbir n-ler üçin Nỳuton – Kotesiň koeffisiỳentleri aşakdaky bahalara eỳedir.

n=1	$H_0 = H_1 = 1/2$
n=2	$H_0 = H_2 = 1/6, \ \ H_1 = 2/3$
n=3	$H_0 = H_3 = 1/8, \ \ H_1 = H_2 = 3/8$
n=4	$H_0 = H_4 = 7/90, \ \ H_1 = H_3 = 16/45, \ H_2 = 2/15$
n=5	$H_0 = H_5 = 19/288, \ \ H_2 = H_3 = 25/144, \ H_1 = H_4 = 25/96$

Trapesiỳalar formulasy.

$$\int_a^b f(x)dx = \frac{6-a}{2n}(y_0 + 2y_1 + ... + 2y_{n-1} + y_n).$$

(3)

(3) – formula bilen kesgitlenen integrallaryň takmyn bahalary hasaplananda, goỳberilỳän hatany bahalandyrmak üçin aşakdaky aňlatma amatlydyr.

$$R = \frac{(b-a)^3}{12n^2} f'''(\xi), a \le \xi \le b. \tag{4}$$

Simpson formulasy.

$$\int_a^b f(x)dx = \frac{b-a}{6n}(y_0 + 4y_1 + 2y_2 + ... + 4y_{2n-1} + y_{2n})$$

(5)

(5) – formula bilen kesgitlenen integrallaryň takmyn bahalaryny hasaplananda gőyberilỳän hatany bahalandyrmak üçin bolsa, aşakdaky aňlatma amatlydyr.

$$R = -\frac{(b-a)^5}{2880n^2} f'''(\xi), a \le \xi \le b.$$

(6)

Mesele .

1.Nỳuton – Kotesiň formulasynyň kőmegi bilen n=4 bolanda berlen integralyň takmyn bahasyny hasaplamaly.

2.Trapesiỳa formulasynyň kőmegi bilen berlen integralyň 0,0001 takyklykda takmyn bahasyny hasaplamaly.

3.Simpson formulasynyň kőmegi bilen 2n=8 bolanda berlen integralyň takmyn bahasyny hasaplamaly.

1) $\int_0^{\pi/2} \frac{\cos x}{1+x} dx$ 2) $\int_{0,7}^{1/3} \frac{1}{\sqrt{2x^2+0,3}} dx$ 3) $\int_{1,2}^{1/6} \frac{\sin(2x-2,1)}{x^2+1} dx.$

1. Hasaplama (2) – formula bilen geçirilỳär, ỳagny

$$J = \int_0^{\pi/2} \frac{\cos x}{1+x} dx = \frac{\pi}{2} \sum_{i=0}^{4} H_i y_{i.}$$

hasaplamalaryň netijeleri aşakdaky tablisada ỳerleşdirilendir.

I	x_i	y_i	H_i	$H_i y_i$
0	0	1	7/90	0,077
1	0,4	0,659	16/45	0,2342
2	0,8	0,393	2/15	0,0523
3	1,2	0,174	16/45	0,0618
4	1,6	0	7/90	0

Diỳmek, $\sum_{i=0}^{4} H_i y_{i.} = 0,4260.$ Onda,

$$J = 1,5707 \cdot 0,4260 = 0,6691$$

.

Hasaplamalaryň personal kompŷuterde alnan netijesi 1-nji programmada gőrkezilendir.

```
cls
rem Nuton-Kotes formulasy
pi=3.141592:a=0:b=pi/2:n=4
h=(b-a)/n
h(0)=7/90:h(1)=16/45:h(2)=2/15:h(3)=16/45:h(4)=7/90
s=0
for i=0 to 4
x=i*h
y(i)=cos(x)/(1+x)
s=s+h(i)*y(i)
next i
s=s*(b-a)
print "Integralyn bahasy s=";s
end

Integralyn bahasy s= .6737464070320129
```

1 – nji programma

2.Hasaplamany berlen taklykda ỳerine ỳetirmek ücin, aşakdaky deňsizligi kanagatlandyrỳan n – i tapalyň

$$\frac{(b-a)^3}{12n^2} m < 0{,}0005,$$

Bu ỳerde

$a = 0{,}7:\ b = 1{,}3.$ $\quad M \geq \max\limits_{x\in[0{,}8;1{,}3]}(f''(x)).$ $\quad f(x) = \dfrac{1}{\sqrt{2x^2+0{,}3}}$ $\quad f(x) = \dfrac{1}{\sqrt{2x^2+0{,}3}}$

funksiỳanyň ikinji tertipdäki önümini alalyň

$$f''(x) = \frac{8x^2 - 0{,}6}{\sqrt{(2x^2 + 0{,}3)^5}}.$$

Diỳmek m=7. Onda ỳokary densizligimiz aşakdaky gőrnuşi alar.

$$\frac{0{,}6^3 \cdot 7}{12n^2} < 0{,}0005$$

Bu ỳerde n^2>256 ỳa-da n>16. Diýmek hasaplamalarda n=20 diýip alalyň.

Hasaplama (3)-formula bilen geçirilỳär, ỳagny

$$J = \int_{0{,}7}^{1{,}3} \frac{1}{\sqrt{2x^2 + 0{,}3}} dx = h\left(\frac{y_0 + y_{20}}{2} + y_1 + \ldots + y_{19} \right).$$

Bu ỳerde $h = (b - a)/n = 0{,}6/20 = 0{,}003.$

$$y_i = y(x_i) = 1/\sqrt{2x_i^2 - 0{,}3},\ x_i = 0{,}7 + ih\ (i = 0{,}1,\ldots,20).$$

Hasaplamalaryň netijeleri aşakdaky tablisada ỳerleşdirilendir.

i	x_i	x_i^2	$2x_i^2 + 0{,}3$	$\sqrt{2x_i^2 + 0{,}3}$	$y_0, y_{20}.$	$y_1, y_2 \ldots, y_{19}$
0	0,7	0,49	1,2	1,1314	0,0883	
1	0,73	0,533	1,3658	1,1686	86	0,85572
2	0,76	0,578	1,4552	1,2063		0,82898
3	0,79	0,624	1,5482	1,2443		0,77973
4	0,82	0,6724	1,6448	1,2825		0,77971
5	0,85	0,7225	1,7450	1,3210		0,75700
6	0,88	0,7744	1,8488	1,3597		0,73546
7	0,91	0,8281	1,9562	1,3986		0,71501

8	0,94	0,8836	2,0672	1,4378		0,69551
9	0,97	0,9409	2,1818	1,4771		0,67700
10	1,00	1.0000	2,3000	1,5166		0,65937
11	1,03	1,0609	2,4018	1,5562		0,64259
12	1,06	1,1236	2,5472	1,5960		0,62657
13	1,09	1,1881	2,6762	1,6356		0,61140
14	1,12	1,2544	2,8088	1,6759		0,59669
15	1,15	1,3225	2,9450	1,7161		0,58272
16	1,18	1,3924	3,0848	1,6564		0,56995
17	1,21	1,4641	3,2282	1,7967		0,55658
18	1,24	1,5376	3,3752	1,8372		0,54431
19	1,27	1,6129	3,5258	1,8777		0,53253
20	1,30	1,6900	3,6800	1,9187		
					0,52129	
				1,40515		12,77022

Diỳmek, $J = 0{,}03\left(\frac{1{,}40515}{2} + 12{,}77022\right) = 0{,}40418 \approx 0{,}404.$

Hasaplamalaryň personal kompŷuterde alnan netijesi 2-nji programmada gőrkezilendir.

```
cls
rem Trapesiyalar formulasy
a=0.7:b=1.30:n=20
h=(b-a)/n
s=1/sqr(2*a^2+0.3)+1/sqr(2*b^2+0.3)
for x=0.73 to 1.27 step h
s=s+2/sqr(2*x^2+0.3)
next x
s=s*h/2
```

```
print "Integralyn bahasy s=";s
end
```

Integralyn bahasy s= .4041787683963776

2 – nji programma

3.Hasaplama (5)-formula bilen geçirilỳär, yagny

$$J = \frac{h}{2}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + 2y_6 + 4y_7 + y_8\right),$$

bu ỳerde

$$y_i = y(x_i) = \frac{\sin(2x_i - 2,1)}{x_i^2 + 1}; x_i = 1,2 + ih, \ (i = 0,1,...,8),$$

$$h = (b-a)/2n = (1,6 - 1,2)/8 = 0,05.$$

Hasaplamanyň netijeleri aşakdaky tablisada ỳerleşdirilendir.

I	x_i	$2x_i - 2,1$	$\sin(2x_i - 2.1)$	$x_i^2 + l$	y_0, y_1	y_1, y_3, y_5, y_7	y_2, y_4, y_6
0	1,20	0,30	0,2955	2,4400	0,1211		
1	1,25	0,40	0,3894	2,5625		0,1520	
2	1,30	0,50	0,4794	2,6900			
3	1,35	0,60	0,5646	2,8225		0,2000	0,1782
4	1,40	0,70	0,6442	2,9600			
5	1,45	0,80	0,7274	3,1024		0,2312	0,2176
6	1,50	0,90	0,7833	3,2500			
7	1,55	1,00	0,8415	3,4025	0,2503	0,2473	0,2410
8	1,60	1,10	0,8912	3,5600			
					0,3714	0,8305	0,6368

$$J = \frac{0,05}{3}\left(0,3714 + 4 \cdot 0,8305 + 2 \cdot 0,6368\right) \approx 0,08278.$$

Hasaplamalaryň personal kompŷuterde alnan netije 3-nji programmada gőrkezilendir.

```
cls
rem Simpson formulasy
a=1.2:b=1.6:n=8:c=1
h=(b-a)/n
s=sin(2*a-2.1)/(a^2+1)+sin(2*b-2.1)/(b^2+1)
for i=1 to 7
x=a+i*h
s=s+(3+c)*sin(2*x-2.1)/(x^2+1)
c=-c
next i
s=s*h/3
print "Integralyn bahasy s=";s
end
```

Integralyn bahasy s= 8.279035985469818E-002

3 – nji programma

Aşakda sanly differensirleme we integrirleme üçin APP görkezilendir.

```
cls
REM Differensirlemanin den aralyk ucin
formulalary
input n,h,z
```

```
dim x(n),y(n), y1(n), y2(n),y3(n),t(n)
if n<3 then print "n<3 yagdayda ulanmaklyk
amatsyz":goto 100
x(0)=0.8:x(1)=1.2:x(2)=1.6:x(3)=2:x(4)=2.4
x(5)=2.8:x(6)=3.2:x(7)=3.6
y(0)=2.857:y(1)=3.946:y(2)=4.938:y(3)=5.801:y(
4)=6.503
y(5)=7.01:y(6)=7.288:y(7)=7.301
for i=0 to n-1
y1(i)=y(i+1)-y(i)
next i
for i=0 to n-2
y2(i)=y1(i+1)-y1(i)
next i
for i=0 to n-3
y3(i)=y2(i+1)-y2(i)
next i
if z<x(2) then 200
if z>x(n-2) then 300
for k=3 to n-3
t(k)=abs(x(k)-z)
next k
j=3:min=t(3)
for k=4 to n-3
if t(k)<min then min=t(k):j=k
next k
q=(z-x(j))/h
if abs(q)<=0.25 then 400
if (0.25<=abs(q))and(abs(q)<=0.75) then 500
if z>x(j) then 600
f1=(y1(j-1)+(2*q+1)*y2(j-1)/2+(3*q^2-1)*y3(j-
2)/6)/h
f2=(y2(j-1)+q*y3(j-2))/h^2
print " Has. G2_FBAA":goto 700
600 f1=(y1(j)+(2*q-1)*y2(j-1)/2+(3*q^2-
1)*y3(j-1)/6)/h
f2=(y2(j-1)+q*y3(j-1))/h^2
```

```
print " Has.  G1_FBAA": goto 700
500 f1=(y1(j)+(2*q-1)*(y2(j-
1)+y2(j))/4+(3*q^2-3*q+0.5)*y3(j-1)/6)/h
f2=(q*(y2(j-1)+y2(j))/2+(2*q-1)*y3(j-1)/2)/h^2
print " Has. B_FBAA": goto 700
400 f1=((y1(j)+y1(j-1))/2+q*y2(j-1)+(3*q^2-
1)*(y3(j-1)+y3(j-2))/12)/h
f2=(y2(j-1)+q*(y3(j-1)+y3(j-2))/2)/h^2
print "  Has. S_FBAA": goto 700
300 if z>x(n) then j=n else if z>x(n-1) then
j=n-1 else j=n-2
q=(z-x(j))/h
f1=(y1(j-1)+(2*q+1)*y2(j-
2)/2+(3*q^2+6*q+2)*y3(j-3)/6)/h
f2=(y2(j-2)+(q+1)*y3(j-3))/h^2
print " Has. N2_FBAA":goto 700
200 if z<x(0) then j=0 else if z<x(1) then j=1
else j=2
q=(z-x(j))/h
f1=(y1(j)+(2*q-1)*y2(j)/2+(3*q^2-
6*q+2)*y3(j)/6)/h
f2=(y2(j)+(q-1)*y3(j))/h^2
print " Has. N1_FBAA"
700 print
print "Meselanin jogaby:    "
print
print "x=";z;" bolanda f1 = ";f1;"   f2= ";f2
100 end
```

```
cls
n=10:a=1.5:b=2.3
h=(b-a)/n
j=0:x=a
for i=0 to n-1
```

```
x=a+i*h:gosub 1000
j=j+h*f:next i
print "Cep gon. j="j
j=0:x=a
for i=1 to n
x=a+i*h:gosub 1000
j=j+h*f:next i
print "Sag gon. j="j
j=0:x=a
for i=0 to n-1
x=a+i*h:x=x+h/2:gosub 1000
j=j+h*f:next i
print "Orta gon. j="j
j=0:x=a
gosub 1000:j=j+f:x=b:gosub 1000:j=j+f
for i=1 to n-1
x=a+i*h:gosub 1000
j=j+2*f:next i
j=j*h/2
print "Trapesiya usuly j="j
j=0:x=a:c=1
gosub 1000:j=j+f:x=b:gosub 1000:j=j+f
for i=1 to n-1
x=a+i*h:gosub 1000
j=j+(3+c)*f:c=-c:next i
j=j*h/3
print "Simpson usuly j="j:stop
1000 f=sqr(0.3*x+1.2)/(1.6*x+sqr(x*x+0.5))
return
end
```

6. Amaly programmalar paketiniň ady differensial deňlemeler ulgamy üçin düzüluși.

Goý n-nji tertipli differensal deňleme berlen bolsun

$$y^{(n)} = f\left(x, y, y', \ldots, y^{(n-1)}\right) \quad (1)$$

Bu differensal deňleme başlangyç şerti kanagatlandyrsyn

$$y(x_o) = y_o, y'(x_o) = y'_o, \ldots, y^{(n-1)}(x_o) = y_o^{(n-1)} \quad (2)$$

bu ýerde $x_o, y_o, y'_o, \ldots, y_o^{(n-1)}$ -berlen sanlar.

(1) – (2) bilelikde ady differensal deňleme üçin Koşi meselesi diýilýär. Koşi meselesiniň çözüwini tapmaklyk y(x) funksiýanyň (1) - (2) deňlemeleri kanagatlandyrýan bahasyny tapmaklyga aýdylýar. Ady differensal deňleme üçin Koşi meselesini çözmekligiň birnäçe san usullary bar. Olar bir ädimli we köpädimli usullara bölünýärler.Eger y_{k+1} bahany tapmaklyk üçin diňe bir y_k baha ulanylsa, onda ol san usulyna birädimli san usuly diýilýär.

1) Eýler ususly

$$\begin{aligned} y' &= f(x, y) \\ y(x_o) &= y_o \end{aligned} \quad (3)$$

(3) Koşi meselesiniň çözüwini tapmak üçin $y_k=y(x_k)$ bahalar tablisasyny gurmaly, bu ýerde

$$\begin{aligned} x_k &= x_o + kh, k = 0,1,2,\ldots, n \\ h &= (b-a)/n \end{aligned}$$

[a,b] çözüwi gözlenýän kesim y_{k+1} baha

$$y_{k+1} = y_k + hf(x_k, y_k) \quad (4)$$

k=0,1,2,…,n-1

formula arkaly hasaplanýar.

2) Eýler-Koşi usuly

ilki bilen $\tilde{y}_{k+1}$ baha hasaplanýar.

$$\tilde{y}_{k+1} = y_k + hf(x_k, y_k)$$

Onsoň

$$y_{k+1} = y_k + \frac{h}{2}\left[f(x_k, y_k) + f(x_{k+1}, \tilde{y}_{k+1})\right] \quad (5)$$

formula arkaly hasaplanýar.

3) Runge-Kutte usuly

Her bir ädimde hasaplamalar aşakdaky formulalar arkaly amala aşyrylýar.

$$y_{i+1} = y_i + \frac{1}{6}\left(k_1^{(i)} + 2k_2^{(i)} + 2k_3^{(i)} + k_4^{(i)}\right) \quad (4)$$

bu ýerde

$$k_1^{(i)} = hf(x_i, y_i)$$

$$k_2^{(i)} = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1^{(i)}}{2}\right)$$

$$k_3^{(i)} = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_2^{(i)}}{2}\right) \qquad x_i = x_o + ih \quad (i=0,1,2,..,n)$$

$$k_4^{(i)} = hf\left(x_i + h, y_i + k_3^{(i)}\right)$$

4)Adams usuly

Birinji tertipli tapawut formulasy

$$y_{k+1} = y_k + q_k + \frac{1}{2}\Delta q_{k-1}$$
$$q_k = hf(x_k, y_k), k = 1,2,... \quad (7)$$

Ikinji tertipli tapawut formulasy

$$y_{k+1} = y_k + q_k + \frac{1}{2}\Delta q_{k-1} + \frac{5}{12}\Delta^2 q_{k-2} \quad (8)$$

Üçinji tertipli tapawut formulasy

$$y_{k+1} = y_k + q_k + \frac{1}{2}\Delta q_{k-1} + \frac{5}{12}\Delta^2 q_{k-2} + \frac{3}{8}\Delta^3 q_{k-3} \quad (9)$$

(9) formulany ulanmak üçin y_o, y_1, y_2, y_3 başlangyç bahalary gerek bolup durýar. Bu bahalary başga san usullary bilen hasaplanýar.

5)Miln usuly

Goý Koşi meselesiniň y_o başlangyç bahasyndan başga $y(x_i) = y_i$ funksiýanyň $x_i = x_o + ih$ (i=1,2,3) nokatlarda belli bolsun. y_i i=4,5,… bahalary tapmak üçin ilki bilen 1-nji Miln formulasy arkaly

$$y_i^{deslapky} = y_{i-4} + \frac{4h}{3}(2f_{i-3} - f_{i-2} + 2f_{i-1})$$

y_i^{des} bahany ulanyp $f_i^{des} = f(x_i, y_i^{des})$ tapyp ikinji Miln formulasy arkaly takyklanýar.

$$y_i^{tak} = y_{i-2} + \frac{h}{3}(f_{i-2} + 4f_{i-1} + f_i^{des})$$

Apsolýut hata $\varepsilon \approx \frac{1}{29}\left|y_i^{tak} - y_i^{des}\right|$ tapylýar we takyklyk ýeterlik bolsa, onda $y_i \approx y_i^{tak}$ alynýar.

```
CLS
Input n,h
Inpu x,y
Print x,y
yy=y
For i=1 to n
Gosub 100
k1=h*f
y=yy+k1/2
x=x+0.5*h
Gosub 100
k2=h*f
y=yy+k2/2
Gosub 100
k3=h*f
x=x+h/2
y=yy+k3
Gosub 100
k4=h*f
y=yy+(k1+2*k2+2*k3+k4)
Print x,y
Next i
Stop
100 Rem “BPF”
f=x+sin(y/2.25)
Return
End
```

Goý Koşi meselesi

$$\begin{cases} y^{(n)} = f\left(x, y, y', ..., y^{(n-1)}\right) \\ y\Big|_{x=x_o} = y_o, y'\Big|_{x=x_o} = y_{o1}, ..., y^{(n-1)}\Big|_{x=x_o} = y_{o(n-1)} \end{cases} \qquad (10)$$

görnüşde berlen bolsun.
Onda (2)-de

$$y' = u_1$$

$$y'' == u_1' = u_2$$

$$y''' = u_1'' = u_2' = u_3$$

....

$$y^{(n)} = ... = u_n$$

Ornuna goýmany ulansak, n-nji tertipli differensal deňlemäni n-sany differensal deňlemeler ulgamy bilen çalşyryp bolýar. (2) deňlemäniň sag tarapy hem n sany bolar, olary degişlilikde $f_1, f_2, ..., f_n$ bilen belgilesek, Onda n sany deňlemleer ulgamyny alarys

$$\begin{cases} u_i = f_i(x, y, u_1, u_2, ..., u_{n-1}) \\ u_o = y_o, u_j = y_j \end{cases} \quad j=1,2,...,n-1,\ i=1,2,...,n$$

Diýmek Runge-Kutta ususlynyň algoritmini ýokary tertipli differensal deňleme üçin ýa-da birinji tertipli differensal deňlemeler ulgamy üçin hem ulanmak bolar.

Koşi meselesini çözmekligiň köpädimli san usullary

$$y' = f(x, y)$$

$$y(x_o) = y_o$$

Koşi meselesiniň çözüwini köpädimli san usullary tapmak gerek bolsun.Onuň üçin Adams we Miln usullarynyň O(h^3) takyklykdaky çözüwini alyp bolýan formulasyny alalyň

$$y_{k+1} = y_k + q_k + \frac{1}{2}\Delta q_{k-1} + \frac{5}{12}\Delta^2 q_{k-2} + \frac{3}{8}\Delta^3 q_{k-3} \quad (11)$$

bu birinji formulada y_o başlangyç bahadanbaşgada y_1, y_2, y_3 çözüwler başga bir san usullary bilen hasaplamalay. (11) formuladaky

$q_k = hf(x_k, y_k)$ deňdir. $\Delta q_k, \Delta^2 q_k, \Delta^3 q_k$ -tükenikli tapawutlar. Onda (11) formula boýunça hasaplama tablisasyny alalyň

K	X_k	Y_k	f_k	q_k	Δq_k	$\Delta^2 q_k$	$\Delta^3 q_k$
0	X_o	Y_o	$F(x_o,y_o)$	q_o	Δq_o	$\Delta^2 q_o$	$\Delta^3 q_o$
1	X_1	Y_1	$F(x_1,y_1)$	Q_1	Δq_1	$\Delta^2 q_1$	....
2	X_2	Y_2	$F(x_2,y_2)$	Q_2	Δq_2	....	
3	X_3	Y_3	$F(x_3,y_3)$	Q_3	....		
4	X_4	Y_4	....	....			
5	X_5	....					
...	....						

$$y_4 = y_3 + q_3 + \frac{1}{2}\Delta q_2 + \frac{5}{12}\Delta^2 q_1 + \frac{3}{8}\Delta^3 q_o$$

Mysal

$$\begin{cases} y' = x + \sin(y/2.25) \\ y(1.14) = 2.2 \end{cases} \qquad (12)$$

[1.4,2.4], h=0.1

Koşi meselesi berlen

$$y_{k+1} = y_k + q_k + \frac{1}{2}\Delta q_{k-1} + \frac{5}{12}\Delta^2 q_{k-2} + \frac{3}{8}\Delta^3 q_{k-3} \qquad (1)$$

k=3,4,5,…

```
Cls
Input n,x(0),h
Input y(0),y(1),y(2),y(3)
Dim x(n),y(n),f(n),q(n),dq(n),ddq(n),dddq(n)
For k=1 to n
```

```
x(k)=x(0)+k*h
Next k
For k=o to 3
Gosub 100
q(k)=h*f(k)
Next k
For k=0 to 2
dq(k)=d(k+1)-d(k)
next k
for k=1 to n
ddq(k)=dq(k+1)-dq(k)
Next k
dddq(0)=ddq(1)-ddq(0)
for k=3 to n-1
gosub 200
Next k
Stop
100 f(k)=x(k)+sin(y(k)/2.25)
Return
End
200 y(k+1)=y(k)+q(k)+dq(k-1)/2+5*ddq(k-1)/12+3*dddq(k-3)/8
Print x(k+1),y(k+1)
j=k+1
gosub 100
q(j)=h*f(j)
dq(j-1)=q(j)-g(j-1)
ddq(j-2)=dq(j-1)-dq(j-2)
ddq(j-3)=ddq(j-2)-ddq(j-1)
Return
End
```

Miln formulasy

$$yd = y_{i-4} + \frac{4h}{3}\left(2f_{i-3} - f_{i-2} + 2f_{i-1}\right)$$

$$f_{id} = f\left(x_i, yd\right)$$

$$yt = y_{i-2} + \frac{h}{3}\left(f_{i-2} - 4f_{i-1} + f_{id}\right), y_i = yt$$

i=4,5,…

```
CLS
Input n,h,x(0)
Input y(0),y(1),y(2),y(3)
Dim x(n),y(n)
For k=1 to n
x(k)=x(0)+k*h
Next k
For k=0 to 3
Gosub 100
Next k
For i=4 to n
yd=y(i-4)+4*h*(2*f(i-3)-f(i-2)+2*f(i-1))/3
y(i)=yd
Gosub 100
yt=y(i-2) +h*(f(i-2)-4*f(i-1)+f(i))/3
y(i)=yt
Gosub 100
AH=ABS(yt-yd)/29
Print Ah
Print x(i),y(i)
Next i
Stop
100 f(i)=x(i)+sin(y(i)/2.25)
Return
 End
```

```
Input x,y,h
For i=1 to 10
Gosub 100
f1=f:x=x+h:y1=y:y=y1+h*f1
gosub 100
y=y1+h*(f1+f)/2
Print x,y
Next i:Stop
100 Rem b/p
f=x+sin(y/2.25)
Return
END
```

7. Amaly programmalar paketiniň hususy baha we hususy wektorlary tapmaklyk üçin düzülüşi.

Berlen A matrisanyň hususy bahalaryny tapmaklyk onuň

$$\det(A - E\lambda) = 0 \qquad (1)$$

häsiýetlendiriji deňlemesiniň köklerini tapmaklykdan ybaratdyr. (1)-nji formuladaky

$$D(\lambda) = \begin{vmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & \cdots & \cdots & a_{nn}-\lambda \end{vmatrix} = 0 \qquad (2)$$

kesgitleýjä A matrisanyň ***häsiýetlendiriji köpagzasy*** diýilýär. Her bir λ_i bahalara Y_i , ýagny

$y_{i1}, y_{i2}, \cdots, y_{in}$ komponentali hususy wektorlar degişlidir. Olar $AY_i = \lambda_i Y_i$ deňlemäni kanagatlandyrýarlar. $\det(A - \lambda E) = 0$ deňlemäniň köklerini tapmak usuly ulanýarlar:
1) häsiýetlendirijini özgertdip häsiýetlendiriji deňlemäniň köklerini tapmaklygyň san usullary ;
2) häsiýetlendiriji deňlemesiniň köklerini adaty usullar bilen tapmaklyk.

Häsiýetlendiriji köpagzanyň koeffisiýentlerini tapmaklygyň

a)Danilewskiý usuly

Danilewskiýnyň usulynyň düýip mazmuny $D(\lambda)$ kesgitleýjini aşakdaky görnüşine getirmekden ybaratdyr

$$D(\lambda) = \begin{vmatrix} p_1 - \lambda & p_2 & p_3 & \cdots & p_n \\ 1 & -\lambda & 0 & \cdots & 0 \\ 0 & 1 & -\lambda & \cdots & 0 \\ \cdots & \cdots & \cdots & & \cdots \\ 0 & 0 & 0 & \cdots & -\lambda \end{vmatrix} \qquad (3)$$

Onda 1-nji setiriň elementleri boýunça dargadyp alarys:

$$D(\lambda) = (p_1 - \lambda)(-\lambda)^{n-1} - p_2(-\lambda)^{n-2} + p_3(-\lambda)^{n-3} - \cdots + (-1)^{n-1} p_n$$

ýa-da

$$D(\lambda) = (-1)^n (\lambda^n - p_1\lambda^{n-1} - p_2\lambda^{n-2} - p_3\lambda^{n-3} - \cdots - p_n) \qquad (4)$$

Indi bolsa (3) görnüşiň alnyşyna seredeliň. Goý

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \qquad P=\begin{bmatrix} p_1 & p_2 & \cdots & p_{n-1} & p_n \\ 1 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

Bu matrisalaryň häsiýetlendiriji köpagzalary birmeňzeşdirler

$$\det(A-\lambda E)=\det(P-\lambda E) \qquad (5)$$

A matrisadan P matrisa geçmeklik n-1 sany birmeňzeş özgertmeler yzygiderligini geçirmekden ybaratdyr. A matrisanyň iň soňky setirini

$$a_{n1}a_{n2} \quad \cdots a_{n,n-1}a_{nn}$$

0 0 ... 0 1 0 görnüşe getirilişine seredeliň. Onuň üçin $a_{n,n-1}\neq 0$ guman edeliň. Eger-de şert ýerine ýetmese, onda A matrisany özgertmeli. A matrisanyň (n-1)-nji sütüniniň hemme elementlerini $a_{n,n-1}$ bölmeli. Onda onyň n-nji setiri aşakdaky görnüşi alar:

$$a_{n1}\, a_{n2} \quad \cdots 1\, a_{nn}$$

Soňra özgerdilen matrisanyň (n-1)-nji sütünini degişlilikde

$$a_{n1}, a_{n2}, \cdots, a_{nn}$$

sanlara köpeldip galan hemme sütünlerden aýyrmaly. Netijede soňky setiri 0 0...1 0 bolan matrisany alarys. Bu ýönekeý operasiýalary birlik matrisanyň üstünden hem geçirip aşakdaky matrisany alarys:

$$M_{n-1} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ m_{n-1,1} & m_{n-1,2} & \cdots m_{n-1,n-1} & & m_{n-1,n} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

bu ýerde

$$m_{n-1,i} = -\frac{a_{ni}}{a_{n,n-1}} \quad i \neq n-1$$

we

$$m_{n-1,n-1} = -\frac{1}{a_{n,n-1}}$$

Şeýlelikde, ýokardaky görkezilen özgertmelerden soňra aşakdaky matrisany alarys:

$$AM_{n-1} = B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1,n-1} & b_{1,n} \\ b_{21} & b_{22} & \cdots & b_{2,n-1} & b_{2,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ b_{n-1,1} & b_{n-1,2} & \cdots & b_{n-1,n-1} & b_{n-1,n} \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

Matrisalary köpeldip B matrisanyň elementlerini üçin aşakdaky formulalary alarys:

$$b_{ij} = a_{ij} + a_{i,n-1}m_{n-1,j} \qquad 1 \leq i \leq n;\ j \neq n-1$$

$$b_{i,n-1} = a_{i,n-1}m_{n-1,n-1} \qquad 1 \leq i \leq n$$

Ýöne bu düzülen $B=AM_{n-1}$ matrisa A matrisa meňzeş bolmanlygy üçin ters M_{n-1}^{-1} matrisany çepinden B marisa köpeldip alarys:

$$M_{n-1}^{-1}AM_{n-1} = M_{n-1}^{-1}B$$

Ters matrisa bolsa aşakdaky görnüşdedir:

$$M_{n-1}^{-1} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{n,n-1} & a_{n,n} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

Goý, $M_{n-1}^{-1}AM_{n-1} = C$. Onda $C = M_{n-1}^{-1}B$. Onda C matisany alarys:

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1,n-1} & c_{1,n} \\ c_{21} & c_{22} & \cdots & c_{2,n-1} & c_{2,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ c_{n-1,1} & c_{n-1,2} & \cdots & c_{n-1,n-1} & c_{n-1,n} \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

Şeýlelikde, ters M_{n-1}^{-1} matrisany çepinden B marisa köpeltsek, onyň diňe (n-1)-nji setiri üýtgeýär. Bu matrisanyň elementlerini aşakdaky formulalar boýunça tapylýar:

$$c_{ij} = b_{ij} \ , \ 1 \le i \le n$$

we

$$c_{n-1,j} = \sum_{k=1}^{n} a_{nk} b_{kj} \ , \ 1 \le j \le n$$

C matrisa *A* matrisa meňzeşdir. Eger $c_{m-1,n-2} \ne 0$ bolsa, onda *C* matrisanyň üstünde hem ýokardaky operasiýalary gaýtalap alarys:

$$D = M_{n-2}^{-1} C M_{n-2}$$

Şeýlelik bilen bu operasiýalary (n-1) gezek gaýtalap, Fobeniusyň matrisasyny alarys:

$$P = M_1^{-1} \cdots M_{n-2}^{-1} M_{n-1}^{-1} A M_{n-1} M_{n-2} \cdots M_1 .$$

Bu özgertmeleriň biriniň hasaplaplamalaryny aşakdaky shemada görkezmek bolar(n=4):

Setiriň nomeri	M^{-1}	Matrisanyň sütunleri				$\sum$	$\sum'$
		1	2	3	4		
1 2 3 4		a_{11} a_{21} a_{31} a_{41}	a_{12} a_{22} a_{32} a_{42}	a_{13} a_{23} a_{33} a_{43}	a_{14} a_{24} a_{34} a_{44}	d_1 d_2 d_3 d_4	
1	M_3 \ M_3^{-1}	m_{31}	m_{32}	m_{33}**-1**	m_{34}	α_1	
5	a_{41}	b_{11}	b_{12}	b_{13}	b_{14}	β_1	γ_1

6 7 8	a_{42} a_{43} a_{44}	b_{21} b_{31} 0	b_{22} b_{32} 0	b_{23} b_{33} 0	b_{24} b_{34} 0	β_2 β_3 1	γ_2 γ_3 1
7’		c_{31}	c_{32}	c_{33}	c_{34}	β_3'	

b)Krylowyň usuly

Goý D(λ) A matrisanyň häsiýetlendiriji köpagzasy bolsun.

$$D(\lambda) = \det(\lambda E - A) = \lambda^n + p_1\lambda^{n-1} + ... + p_n$$

Onda Gamilton-Keli toždestwasy esasynda alarys

$$D(A) = A^n + p_1 A^{n-1} + ... + p_n E = 0$$

Erkin nuldan tapawutly wektor alalyň

$$y^{(0)} = \begin{bmatrix} y_1^{(0)} \\ . \\ . \\ . \\ . \\ y_n^{(0)} \end{bmatrix}$$

D(A)-ny sagdan $y^{(0)}$ wektora köpeldip alarys

$$A^n y^{(0)} + p_1 A^{n-1} y^{(0)} + \ldots + p_n y^{(0)} = 0$$

Goý $A^k y^{(0)} = y^{(k)}$ bolsun, onda alarys

$$y^{(n)} + p_1 y^{n-1} + \ldots + p_n y^{(0)} = 0$$

ýa-da

$$\begin{bmatrix} y_1^{(n-1)} y_1^{(n-2)} \ldots y_1^{(0)} \\ y_2^{(n-1)} y_2^{(n-2)} \ldots y_2^{(0)} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ y_n^{(n-1)} y_n^{(n-2)} \ldots y_n^{(0)} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \ldots \\ p_n \end{bmatrix} = -\begin{bmatrix} y_1^{(n)} \\ y_2^{(n)} \\ \ldots \\ y_n^{(n)} \end{bmatrix}$$

Şeýlelikde wektor görnüşindäki ýazgy aşakdaky deňlemeler ulgamy bilen ekwiwalentdir

$$p_1 y_j^{(n-1)} + p_2 y_j^{(n-2)} + \ldots + p_n y_j^{(0)} = -y_j^{(n)} (j = 1, 2, \ldots, n)$$

Bu ulgamdan $p_1, p_2, \ldots, p_n$ näbellileri kesgitlemek bolar

$$y^{(k)} = A y^{(k-1)} (k = 1, 2, \ldots, n)$$

Şeýlelik bilen $y^{(k)}$ wektoryň koordinatalaryny $y_1^{(k)}, y_2^{(k)}, \ldots, y_n^{(k)}$

hasaplama formulasyny alarys

$$\begin{cases} y_i^{(1)} = \sum_{j=1}^{n} a_{ij} y_j^{(0)} \\ y_i^{(2)} = \sum a_{ij} y_j^{(1)} \\ \dots\dots\dots\dots\dots\dots\dots \\ y_i^{(n)} = \sum_{j=1}^{n} a_{ij} y_j^{n-1} (i = 1, 2, \dots, n) \end{cases}$$

c)Lewerrýe-Fadeýewiň usuly

Lewerrýe usulynyň esasy manysy A matrisanyň derejelerini hasaplamakdan başlahýar

$$A^k = A^{k-1} \cdot A \quad (k = 1, 2, .., n)$$

Soňra her bir A^k matrisalaryň yzyny hasaplanýar:

$$SpA^k = \lambda_1^k + \lambda_2^k + \dots + \lambda_n^k = \sum_{i=1}^{n} a_{ii}^{(k)}; \quad A^k = [a_{ij}^{(k)}]$$

Häsiýetlendiriji deňlemäniň koeffisiýentlerini aşakdaky formula boýunça kesgitlenýar:

$$kp_k = SpA^k - p_1 SpA^{k-1} - \dots - p_{k-1} SpA$$

Netijede A matrisa üçin aşakdaky häsiýetlendiriji deňlemäni alarys:

$$(-1)^{(n)} D(\lambda) = \lambda^n - p_1 \lambda^{n-1} - p_2 \lambda^{n-2} - \dots - p_n = 0.$$

Fadeýew Lewerrýeniň usulyny üýtgedip aşakdaky usulyny hödürleýär. Ilki bilen matrisalaryň yzygiderligini gurulýar:

$$A_1 = A; \qquad SpA_1 = p_1; \qquad B_1 = A_1 - p_1E;$$

$$A_2 = AB_1; \qquad \frac{1}{2}SpA_2 = p_2; \qquad B_2 = A_2 - p_2E;$$

....

$$A_{n-1} = AB_{n-2}; \qquad \frac{1}{n-1}SpA_{n-1} = p_{n-1}; \quad B_{n-1} = A_{n-1} - p_{n-1}E;$$

$$A_n = AB_{n-1}; \qquad \frac{1}{n}SpA_n = p_n; \qquad B_n = A_n - p_nE;$$

Netijede aşakdaky deňlemäni alarys:

$$\lambda^n - p_1\lambda^{n-1} - p_2\lambda^{n-2} - \cdots - p_n = 0.$$

$B_n=0$ aňlatma bu prosesi barlaýar. Bu usuly $A^{-1} = B_{n-1}/p_n$ tapmaga mümkinçilik berýär. Hususy wektorlary hasaplamak üçin aşakdaky formulalar ulanylýar:

$$\bar{X}_0 = \bar{E}; \quad \bar{X}_i^{(\bar{k})} = \lambda_k X_{i-1}^{(k)} + b_i^{(k)} \qquad (i = 1,2,3,...,n-1),$$

bu ýerde E-birlik matrisanyň sütüni, b_i^k- B_k matrisanyň sütüni. X_{n-1}^k hususy wektor λ_k hususy baha degişlidir.

Matrisanyñ birinji we ikinji hususy bahalary we olaryñ hususy wektorlaryny tapmaklygyň san usullary

a) iterasiýa usuly

$Y_i=AY_{i-1}$ (i=1,2,...) wektor yzygyderligi gurulýar, bu ýerde A-berlen matrisa, Y_0- erkin wektor. Onda birinji hususy baha aşakdaky formula arkaly hasaplanýar

$$\lambda_1 \approx \frac{y_i^{(k+1)}}{y_i^{(k)}},$$

bu ýerde $y_i^{(k)} we\, y_i^{(k+1)}$ – iki sany wektorlar yzygyderliginiñ biratly koordinatlary. Ikinji hususy baha

$$\lambda_2 \approx \frac{y_i^{(k+1)} - \lambda_1 y_i^{(k)}}{y_i^{(k)} - \lambda_1 y_i^{(k-1)}},$$

bu ýerde $y_i^{(k+1)}, y_i^{(k)} we\, y_i^{(k+1)}$ – üç sany wektorlar yzygyderliginiñ biratly koordinatlary.Hususy wektorlar $\overline{X}_1 \approx \overline{Y}_k, \overline{X}_2 \approx \overline{Y}_{k+1} - \lambda_1 \overline{Y}_k$.

b) matrisany derejä göterme usuly

Bu usulda $A, A^2, A^3, ..., A^{2^k}$ matrisalar yzygiderligi gurulýar, onsoñ

$$Y_m = A^m Y_0; Y_{m+1} = AY_m,$$

wektorlar tapylýar, bu ýerde $m=2^k$. Onda alarys

$$\lambda_1 \approx \frac{Y_i^{(m+1)}}{Y_i^{(m)}}; \overline{X}_1 \approx \overline{Y_m} \, (i = 1,2,...).$$

c) skalýar köpeltmek hasyly usuly

Bu usulda iki sany wektorlar yzygiderligi gurulýar:

$$Y_0; Y_1 = AY_0; Y_2 = AY_1; ...; Y_k = AY_{k-1},$$

we

$$Y_0; Y_1' = A'Y_0; Y_2' = A'Y_1'; ...; Y_k' = A'Y_{k-1}',$$

bu ýerde A we A' degişlilikde berlen we transponirlenen matrisalar. Onda alarys

$$\lambda_1 \approx \frac{(Y_k' \cdot Y_k)}{(Y'_{k-1} \cdot Y_k)},$$

eger-de A-matrisa simmetrik görnüşli bolsa, onda

$$\lambda_1 = \frac{(Y_k \cdot Y_k)}{(Y_{k-1} \cdot Y_k)}.$$

Aşakda hususy bahalary we hususy wektorlary tapmaklygyñ programmalar toplumlary gőrkezilendir

```
cls
rem Lewerre-Fadeyewin usuly
print "Matrisanyn tertibini giriz n=";: input
n
dim a(n,n),p(n),aa(n,n),s(n),c(n,n)
```

```
for i=1 to n: for j=1 to n
print "aa("i","j")=";:input
aa(i,j):a(i,j)=aa(i,j)
next j: next i
for m=1 to n
s(m)=0: for i=1 to n
s(m)=s(m)+a(i,i): next i
for k=1 to n:for j=1 to n: s=0
for i=1 to n: s=s+a(k,i)*aa(i,j): next i
c(k,j)=s: next j: next k
for k=1 to n: for j=1 to n
a(k,j)=c(k,j): next j: next k: next m
p(1)=-s(1)
for i=2 to n:sum=0: for k=1 to i-1
sum=sum+p(k)*s(i-k):next k
p(i)=-(s(i)+sum)/i: next i
print
print "Hasiyet. den. koef. bahasy:"
print
for i=1 to n
print "p("i")=";p(i)
next i
end

cls
rem Dereja goterme usuly
print "Matrisanyn tertibini giriz n=": input n
dim
a(n,n),d(n,n),x1(n),x2(n),y(n),yk(n),ykk(n),c(
n),gat(n),gat2(n)
for i=1 to n: for j=1 to n
print "a("i","j")=":input
a(i,j):aa(i,j)=a(i,j)
next j: next i
for i=1 to n
print "y("i")=": input y(i) :yk(i)=y(i):
ykk(i)=y(i): next i
```

```
for d=1 to 4
for k=1 to n: for j=1 to n: s=0
for i=1 to n:
s=s+aa(k,i)*aa(i,j): next i
d(k,j)=s: next j: next k
for i=1 to n: for j=1 to n:aa(i,j)=d(i,j):next
j:next i
next d
for k=1 to n: s=0
for i=1 to n
s=s+aa(k,i)*y(i): next i
c(k)=s: next k
for k=1 to n: s=0
for i=1 to n
s=s+a(k,i)*c(i):next i
y(k)=c(k):yk(k)=s: next k
for k=1 to n: s=0
for i=1 to n
s=s+a(k,i)*yk(i): next i
ykk(k)=s: print ykk(k): next k
for k=1 to n:gat(k)=yk(k)/y(k) :  next k
max=y(1)
 for k=2 to n
if y(k)>max then max=y(k)
 next k
for k=1 to n: x1(k)=y(k)/max: next k
s=0: for k=1 to n
m1=ykk(k)-gat(1)*yk(k):x2(k)=m1
m2=yk(k)-gat(1)*y(k)
gat2(k)=m1/m2
s=s+gat2(k)
 next k
gat2(1)=s/n
max=x2(1)
for k=2 to n: if x2(k) > max then max=x2(k):
next k
for k=1 to n: x2(k)=x2(k)/max: next k
```

```
print "Matrisanyn moduly boyunca in uly 1-nji
hususy bahasy = "gat(1):print
print "Matrisanyn    2- nji   hususy   bahasy
= "gat2(1):print
print "Matrisanyn 1 - nji hususy
wektory":print
print "X1(";:for k= 1 to n
print x1(k)";"; : next k: print ")":print
print "Matrisanyn 2 - nji hususy
wektory":print
print "X2(";:for k=1 to n
print x2(k)";";: next k: print ")"
end
```

```
cls
rem Iterasiya usuly
print "Matrisanyn tertibini giriz n=": input n
dim
a(n,n),x1(n),x2(n),y(n),yk(n),ykk(n),c(n),gat(
n),gat2(n)
for i=1 to n: for j=1 to n
print "a("i","j")=":input a(i,j)
next j: next i
for i=1 to n
print "y("i")=": input y(i) :yk(i)=y(i):
ykk(i)=y(i): next i
j=1
2 j=j+1
for k=1 to n: s=0
for i=1 to n:
s=s+a(k,i)*y(i): next i
c(k)=s: next k
for k=1 to n
```

```
ykk(k)=yk(k)
 yk(k)=y(k)
  y(k)=c(k): next k
for k=1 to n:gat(k)=y(k)/yk(k):  next k
for k=2 to n: if abs(gat(k)-gat(k-1)) >0.0001
then goto 2
next k
max=y(1)
 for k=2 to n
if y(k)>max then max=y(k)
 next k
for k=1 to n: x1(k)=y(k)/max: next k
s=0: for k=1 to n
m1=y(k)-gat(1)*yk(k):x2(k)=m1
m2=yk(k)-gat(1)*ykk(k)
gat2(k)=m1/m2
s=s+gat2(k)
 next k
gat2(1)=s/n
max=x2(1)
for k=2 to n: if x2(k) > max then max=x2(k):
next k
for k=1 to n: x2(k)=x2(k)/max: next k
print "Matrisanyn moduly boyunca in uly 1-nji
hususy bahasy = "gat(1):print
print "Matrisanyn    2- nji   hususy   bahasy
= "gat2(1):print
print "Matrisanyn 1 - nji hususy
wektory":print
print "X1(";:for k= 1 to n
print x1(k)";"; : next k: print ")":print
print "Matrisanyn 2 - nji hususy
wektory":print
print "X2(";:for k=1 to n
print x2(k)";";: next k: print ")" : end
```

```
cls
rem Skalyar kopeltmek hasyly usuly
print "Matrisanyn tertibini giriz n=": input n
dim a(n,n),x(n),y(n),yk(n),c(n),gat(n)
for i=1 to n: for j=1 to n
print "a("i","j")=":input a(i,j)
next j: next i
for i=1 to n
print "y("i")=": input y(i) :yk(i)=y(i): next
i
j=1: gat(1)=0
2 j=j+1
for k=1 to n: s=0
for i=1 to n
s=s+a(k,i)*y(i): next i
c(k)=s: next k
for k=1 to n
 yk(k)=y(k)
  y(k)=c(k): next k
for i= 1 to n: s1=0:s2=0
for k=1 to n
s1=s1+y(k)*y(k):s2=s2+yk(k)*y(k):  next k
next i
gat(j)=s1/s2
 if abs(gat(j)-gat(j-1)) >0.0001 then goto 2
max=y(1)
 for k=2 to n
if y(k)>max then max=y(k)
 next k
for k=1 to n: x(k)=y(k)/max: next k
print "Matrisanyn moduly boyunca in uly 1-nji
husussy bahasy = "gat(j): print
print "Matrisanyn 1 - nji hususy
wektory":print
print "X(";:for k= 1 to n
print x(k)";"; : next k: print ")"
```

```
end
```

```
cls
print "Berlen matrisanyn hasiyetlendiriji kop.
has."
print "Danilewskin usuly bilen cozulisi"
print "Matrisanyn olcegini giriz n=":input n
dim a(n,n)
print "Matrisanyn elementlerini giriz"
for i=1 to n:for j=1 to n
print "A("i","j")=": input a(i,j)
next j:next i: q=1
40 for k=q to n-1:l=0:for i=k+1 to n
if abs(l)-abs(a(i,k))<0 then l=a(i,k): p=i
next i: if l<>0 then 90
for j=q to k: a(0,j)=a(j,k): next j
q=k+1: print "k=";k: goto 40
90 if k+1=p then 140
for j=k to n: r=a(k+1,j)
a(k+1,j)=a(p,j):a(p,j)=r:next j
for j=q to n: r=a(j,k+1)
a(j,k+1)=a(j,p):a(j,p)=r: next j
140 for j=q to n: a(0,j)=a(j,k): next j
c=a(k+1,k): for j=k to n
a(k+1,j)=a(k+1,j)/c:for i=q to n
if i=k+1 then 190
a(i,j)=a(i,j)-a(0,i)*a(k+1,j)
190 next i: next j
for i=q to n: s=0: for j=k+1 to n
s=s+a(i,j)*a(0,j):next j
if i-1<=k then 240
d=0: goto 260
240 if i=q then d=0: goto 260
d=a(0,i-1)
260 a(i,k+1)=s+d: next i: next k
```

```
print "Meselanin jogaby:"
for j=q to n: a(0,j)=a(j,n): next j
i=0:for j=n to 1 step -1:i=i+1: print
"p(";i;")="; a(0,j)
next j: end
cls
rem Krylowyn usuly
print "Matrisanyn tertibini giriz n=": input n
dim a(n,n),b(n),x(n),aa(n,n),y(n),c(n)
for i=1 to n: for j=1 to n
print "aa("i","j")=":input aa(i,j)
next j: next i
for i=1 to n
print "y("i")=": input y(i): a(i,n)=y(i): next
i
for j=1 to n
for k=1 to n: s=0
for i=1 to n: s=s+aa(k,i)*y(i): next i
c(k)=s: next k
for k=1 to n
y(k)=c(k): a(k,n-j)=c(k): next k: next j
for k= 1 to n
b(k)=-a(k,0): next k
REM Gaussyn kompakt shemasy
n1=n-1
for k=1 to n1
if a(k,k)<>0 then goto 4
i=k+1
6 if a(i,k)<>0 then goto 5
i=i+1
if i<=n then goto 6
print "DENlemeler ulgamynyn cozuwi yok":end
5 i=k
9 v=a(k,l):a(k,l)=a(i,l):a(i,l)=v
l=l+1
if l<=n then goto 9
v=b(k):b(k)=b(i):b(i)=v
```

```
4 j1=k+1
for j=j1 to n
a(k,j)=a(k,j)/a(k,k)
next j
b(k)=b(k)/a(k,k)
ki=k+1
for i=ki to n
for j=ki to n
a(i,j)=a(i,j)-a(i,k)*a(k,j)
next j
b(i)=b(i)-a(i,k)*b(k)
next i
next k
x(n)=b(n)/a(n,n)
k=n-1
26 r=0.0
j=n
23 r=r+a(k,j)*x(j)
if j-k>1 then j=j-1: goto 23
x(k)=b(k)-r
if k>1 then k=k-1: goto 26
print
print "Hasiyet. den. koef. bahasy:"
print
for i=1 to n
print "p("i")=";x(i)
next i
end
```

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